The Welfare Cost of Structural Uncertainty

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1. Introduction

□ Background

○ Qualitative and quantitative assessment regarding the welfare cost of economic fluctuations

  — Business cycles may entail welfare costs to risk-averse consumers. (qualitative)

  — How much would be the welfare losses? (quantitative)

○ Stabilization policies such as monetary and fiscal policies may be assessed based on answers to the above questions.
Motivations and main questions of this paper

○ What if the structure behind economic fluctuations were not fixed-known but variable with some probability distribution?
  — The structure may not be fixed but may be characterized by some degree of uncertainty.
  — The uncertainty associated with the structure may be called high-order risk in contrast to the normal risk of economic fluctuations given the structure.

○ How much of additional welfare losses would be incurred by the structural uncertainty?
  — The additional component, the structural uncertainty may entail additional welfare losses.
  — How much would be the additional amount of welfare costs?
Related literature

○ Previous studies on the welfare costs of business cycles
  — Barro (2009): “Rare Disasters, Asset Prices, and Welfare Costs,” AER

○ Previous studies on the structural uncertainty (high-order risk)
2. Analytical framework for macroeconomic welfare analysis

□ Welfare measure
○ The welfare may be measured as the expected present value of the streams of future utilities as (1)

\[
W(C_0; P) = E_0[\sum_{t=0}^{\infty} e^{-\rho t} U(C_t)]
\]

□ Welfare-equivalent transformation
○ The welfare gains/costs may be computed as compensating changes in consumption (\(\lambda\)) to attain the same level of welfare given changes in economic conditions from \(P\) to \(\tilde{P}\) for example.

\[
W((1 + \lambda)C_0; P) = W(C_0; \tilde{P})
\]

— Welfare gains/costs are transformed into and evaluated in the consumption dimension.
Data generating process of consumption

○ The welfare measure may depend on the underlying stochastic process of consumption.

○ Consumption may be modeled by either a trend-stationary or a difference-stationary process.

    — If Consumption followed a deterministic time-trend with an i.i.d. random error (3), consumption would revert to the deterministic time trend in response to shocks to the system.

    \[
    C_t = C_0 e^{\mu t} e^{-\sigma^2/2} \epsilon_t \quad \text{with} \quad \ln \epsilon_t \sim \text{i. i. d. } N(0, \sigma^2)
    \]

    — If Consumption followed a difference-stationary process with an i.i.d. random error (4), shocks to the system would have permanent effects on the consumption level.

    \[
    g_{t,t+1} \equiv \ln \left( \frac{C_{t+1}}{C_t} \right) = \mu + \epsilon_{t+1} \quad \text{with} \quad \epsilon_{t+1} \sim \text{i. i. d. } N(0, V)
    \]

○ Thus, the latter case may have bigger effects on the welfare measure.
The welfare measure may depend on the DGPs and preferences.

— For the trend-stationary DGP & the CRRA preference,

\[ W_1(C_0; P) = \frac{C_0^{1-\gamma}}{(1-\gamma)} m(1 - \gamma) \sum_{t=0}^{\infty} e^{(\mu - \rho)t}, \text{ where } m(1 - \gamma) = E_0[(e^{-\sigma^2/2} \varepsilon_t)^{(1-\gamma)}] \]

— For the difference-stationary DGP & the CRRA preference,

\[ W_2(C_0; P) = \frac{C_0^{1-\gamma}}{(1-\gamma)} \{1 - \beta m(1 - \gamma)\}^{-1}, \text{ where } m(1 - \gamma) = E_0[e^{(1-\gamma)g_{t,t+1}}] \]

— For the difference-stationary DGP & the Epstein-Zin preference,

\[ W_3(C_0; P) = \frac{C_0^{1-\gamma}}{(1-\gamma)} \{1 - \beta(m(1 - \gamma)^{1-\gamma})^{1-\theta}\}^{\frac{1-0}{1-\theta}}, \text{ where } m(1 - \gamma) = E_t[e^{(1-\gamma)g_{t,t+1}}] \]

The welfare measures are affected by the MGFs that are again affected by the DGPs.
Discussion on the uncertainty associated with the structure (V) of the DGP?

○ V is the main structural parameter that generates shocks to the future economic growth rates (4).

○ A point-estimate of V based on past observations (\(\hat{V}\)) may not be enough to tell the true parameter value of the DGP for the future economic growth rates.
  — Econometricians provide an estimate of V with some probability distributions.
  — Consumers within the economic system may also view \(\hat{V}\) with some probability distribution.

○ The economic system may also keep evolving so that the structure of the DGP may be associated with some uncertainty.
  — In-sample point-estimate of V may not be enough for the out-of-sample prediction for the future economic growth rates.
How to deal with uncertainty associated with the structure (V)?

○ If V were assumed constant, the MGF would be computed by (8).

\[
\ln\{m(1 - \gamma)\} = (1 - \gamma)\mu_g + (1 - \gamma)^2\text{Var}(g)
\]

○ If V were not constant but stochastic, the distribution of shocks to consumption would be computed as a mixture of normal random variables:

\[
\varepsilon \sim \int_0^\infty f_{\varepsilon|V}(\varepsilon|V)g(V)dV
\]

— The random V may be assumed to be drawn from the Inverse Gaussian (IG) distribution in order to keep the model tractable as well as to keep the MGFs finite.

— In this case, the MGF would be computed by (9).

\[
\ln\{m(1 - \gamma)\} = (1 - \gamma)\mu_g + \bar{\alpha}\left\{1 - \sqrt{1 - \delta^2(1 - \gamma)^2/\bar{\alpha}^2}\right\}
\]
Changes in economic conditions— from \( m(\cdot) \) to \( \tilde{m}(\cdot) \) in notations of MGFs — may result in welfare gains/costs, which may be computed by solving the welfare-equivalent transformation (2).

The welfare gains/costs \((\tilde{\lambda})\) may depend on the DGPs and preferences.

— For the trend-stationary DGP & the CRRA preference,

\[
\tilde{\lambda}_1 = \left\{ \frac{m(1-\gamma)}{\tilde{m}(1-\gamma)} \right\}^{1/(\gamma-1)} - 1
\]

— For the difference-stationary DGP & the CRRA preference,

\[
\tilde{\lambda}_2 = \left\{ \frac{1-\beta}{1-\beta} \frac{\tilde{m}(1-\gamma)}{m(1-\gamma)} \right\}^{1/(\gamma-1)} - 1
\]

— For the difference-stationary DGP & the Epstein-Zin preference,

\[
\tilde{\lambda}_3 = \left\{ \frac{1-\beta(1-\gamma)}{1-\beta(\tilde{m}(1-\gamma))^{(1-\theta)/(1-\gamma)}} \right\}^{1/(1-\theta)} - 1
\]

* Note that \( \tilde{\lambda}_3 \) is the same as \( \tilde{\lambda}_2 \) if \( \theta = \gamma \), the condition at which the EZW preference is the same as the CRRA preference.
3. Welfare analysis under the CRRA preference

- Welfare gains/costs of alternative scenarios may be computed in reference to the baseline.
  - Baseline scenario may be taken as the case of consumption fluctuation with the structure V fixed at the sample estimate.
    - The baseline scenario of the DGP may be represented by the MGF \( m(\cdot) \).
  - Alternative scenarios are either
    - the case with V constant and much smaller or
    - the case with V risky (stochastic)
    - These alternatives can be described by the MGF \( \tilde{m}(\cdot) \).
  - Welfare gains/costs of alternative scenarios can be computed by (11), (12), and (13) given \( m(\cdot) \) and \( \tilde{m}(\cdot) \).
Welfare gains from eliminating all consumption fluctuations given the CRRA preference and the trend-stationary DGP.

The welfare gain is computed as $e^{\gamma \sigma^2/2} - 1 \approx \gamma \sigma^2/2$.

- The welfare gain would be about 0.002 (0.2%) if $\gamma = 4$ and $\sigma^2 = 0.032^2$ were assumed as benchmark parameter values for example.

- This is the very result based on which Lucas (2003) argued that welfare gains from demand side stabilization policy could be trivial.

- However, the result may be due to the misspecification of consumption fluctuations as Barro (2009) argued; thus, it may be worthwhile to analyze the welfare effects of consumption fluctuations under alternative specifications.
Welfare gains from eliminating all consumption fluctuations given the CRRA preference and the difference-stationary DGP

The welfare gain is computed as 1.16% for the benchmark parameter values shown in Table 1.

Table 1: Benchmark parameter values

<table>
<thead>
<tr>
<th>Main parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of time preference per year (ρ)</td>
<td>0.052</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion (γ)</td>
<td>4</td>
</tr>
<tr>
<td>Mean annual consumption growth rate (μₖ)</td>
<td>0.025</td>
</tr>
<tr>
<td>S.D. of annual consumption growth rate (σ)</td>
<td>0.032</td>
</tr>
<tr>
<td>Elasticity of Inter-temporal Substitution (ψ)</td>
<td>2</td>
</tr>
</tbody>
</table>

Notes: Lucas (2003) and Barro (2009) may be referred to for the parameter values.

The welfare gain is much bigger for the difference-stationary DGP than for the trend-stationary DGP because shocks to the consumption growth have permanent effects on the consumption level in the former case.
Welfare costs from incorporating the structural uncertainty (stochastic V) given the CRRA preference and the difference-stationary DGP

- The welfare cost is computed as -0.24% for the benchmark parameter values given in Table 1.
- This result implies that consumers with the CRRA preference may be willing to cost up to 0.24% of the current consumption in return for eliminating only the structural uncertainty in the difference-stationary consumption process.
Welfare gains/costs for different degrees of risk aversion given the CRRA preference and the difference-stationary DGP

- The welfare gains from eliminating consumption fluctuation may increase from 1.16% to 3.68% as the risk aversion $\gamma$ increases from 4 to 10.
- The welfare cost due to the structural uncertainty may increase from 0.24% to 1.54% as $\gamma$ increases from 4 to 10.
- The welfare costs become bigger as more structural uncertainties are incorporated into the DGP of consumption.
Figure 1: Welfare gains/costs under the CRRA preference

Notes: $\alpha = \gamma$ is the case with the maximum structural uncertainty based on the NIG distribution. 0*SD(g), 0.5*SD(g), and 0.75*SD(g) are welfare gains from reducing the standard deviation of consumption growth SD(g) to 0%, 50%, and 75% respectively.
4. Welfare analysis under the EZW preference

- Welfare gains from eliminating all consumption fluctuations given the Epstein-Zin preference and the difference-stationary DGP
  - The welfare gain is computed as 3.85% for the benchmark parameter values shown in Table 1.
  - Thus, consumers with the EZW preference may be willing to give up about 3.85% of the current consumption in order to avoid the consumption fluctuations in the difference-stationary DGP.

- Welfare costs from incorporating the structural uncertainty (stochastic $V$) given the Epstein-Zin preference and the difference-stationary DGP
  - The welfare cost is computed as -0.76% for the benchmark parameter values given in Table 1.
  - Thus, consumers with the EZW preference may be willing to cost up to 0.76% of the current consumption in return for eliminating only the structural uncertainty in the difference-stationary consumption process.
- Welfare gains/costs for different degrees of risk aversion given the Epstein-Zin preference and the difference-stationary DGP

  - The welfare gains from eliminating consumption fluctuation may increase from 3.85% to 11.75% as the risk aversion $\gamma$ increases from 4 to 10.

  - The welfare cost due to the structural uncertainty may increase from 0.76% to 4.11% as $\gamma$ increases from 4 to 10.

  - The welfare costs become bigger as more structural uncertainties are incorporated into the DGP of consumption.
Figure 2: Welfare gains/costs under the EZW preference

Notes: $\alpha = \gamma$ is the case with the maximum structural uncertainty based on the NIG distribution. $0\cdot SD(g)$, $0.5\cdot SD(g)$, and $0.75\cdot SD(g)$ are welfare gains from reducing the standard deviation of consumption growth $SD(g)$ to 0%, 50%, and 75% respectively.
5. Conclusion

☐ This paper derived the welfare gains/costs in terms of the MGFs for the Epstein-Zin preference as well as the CRRA preference.

☐ Consumption fluctuations may entail non-trivial welfare costs when consumption is represented by the difference-stationary DGP.

☐ The welfare costs may be bigger when structural uncertainty is taken into account in the welfare assessment.

○ In addition, the more risk-averse consumers are, the bigger the impact of the structural uncertainty would be on the welfare cost.