The Middle Generations Saves More: Analytical Approach with Habit Formation in Three Generation Overlapping Generations Economy

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On Savings and Habit Formation in an Overlapping-Generations Economy

Abstract

In this work, we extend Constantinides, Donaldson and Mehra's work (2002) by incorporating habit formation in a three period overlapping-generations economy. Using comparative static analysis and model calibrations, we verify that there is a positive impact of habit formation on the savings levels of middle-aged investors. The findings imply that taking into account habit formation within the overlapping-generations framework provides a more convincing explanation of the equity premium puzzle than what would be obtained from a non-habit forming model.

JEL Classification: D0; E0; E2; G0

Key words: Habit formation utility; Overlapping-generations model; Savings; Equity premium

1. Introduction

Habit formation has been widely used in recent studies of financial economics as an important motive to explain the savings and portfolio decision. For example, Constantinides (1990) and Abel (1990) show that habit-forming consumption, with its flexibility in modeling risk aversion and consumption paths, can partially resolve the equity premium puzzle posed by Mehra and Prescott (1985).[1] This finding has indeed motivated a line of habit formation approaches in dynamic modeling of optimal savings and portfolio decisions (e.g., Sundaresan, 1989; Jermann, 1998; Campbell and Cochrane, 1999; and Lettau and Uhlig, 2000).

[1] See Cochrane and Hansen (1992) and Kocherlakota (1996) for surveys on the equity premium puzzle.

On the other hand, in an attempt to resolve the premium puzzle, Constantinides, Donaldson, and Mehra (2002) (hereafter CDM) propose an overlapping-generations (hereafter OLG) model that explicitly captures the saving and dissaving behavior of consumers subject to a borrowing constraint. CDM show that, with a simple time separable Constant Relative Risk Averse (hearafter CRRA) utility function and a borrowing constraint, consumers in a three-period overlapping-generations economy have an incentive to hold a diversified portfolio for different stages over their life cycle. The key feature of the CDM's OLG approach is that, the inability of the young-aged generation to hold equity together with high fluctuations of equity income reduce the demand for equity and, consequently, raise the net demand for bonds by the middle-aged (generation) investors. Thus, the middle-aged investors' savings decision has a substantial impact on the equity premium (*i.e*., a low risk-free rate and a high risk premium).

In this paper, we extend CDM's work by incorporating habit formation into the OLG economy, such that the habit-forming consumers' optimal savings decision is derived from an overlapping-generations framework.[2] Using comparative statics and model calibration, we verify a positive impact of habit formation on the middle-aged investors' savings level under a reasonable parameter set. The main findings imply that incorporating habit formation in an overlapping-generations framework yield more plausible results on the role of the middle-aged investors' savings condition (i.e., higher incentives to save) than the results generated by CDM.[3]

[2] As far as we know, researchers have not incorporated habit formation in a three-period overlapping-generations model. For example, Constantinides (1990) and Abel (1990) suggest the habit formation utility in an infinite horizon model, while Constantinides, Donaldson and Mehra (2002) propose the CRRA utility in an overlapping-generations economy.

[3] CDM (2002) assume a representative consumer with the CRRA utility.

In CDM's framework, a borrowing constraint prevents the young-aged generation from holding equity, such that equity prices are assumed to be exclusively determined by the middle-aged investors. Knowing that their future retirement income is either zero or deterministic and that their future consumption is highly correlated with equity income, the middle-aged investors will save more by holding more bonds and less equity. Therefore, the middle-aged investors' savings decision has a dominant impact on the level of the equity premium and the bond yield.

Under the habit formation utility and the OLG economy, however, the impact of the middle-aged investors' savings decision on the demand for equity and bonds is affected by not only the presence of a borrowing constraint but also by the habit formation process. With a stronger incentive to secure their future consumption, the habit-forming middle-aged investors will save even more than do the middle-aged investors in the CDM's case (with non-habit formation utility).[4] Thus, habit formation causes even higher demand for bonds (yielding a lower risk-free rate) and less demand for equity (yielding a higher required return for holding equity), thereby yielding a higher equity premium than does the CDM's framework. This implies that incorporating habit formation within the OLG framework can provide more satisfactory explanations on the equity premium puzzle.[5]

[4] The Relative Risk Averse (hearafter RRA) coefficient of the habit formation utility is greater than that of the CRRA utility, so that a habit-forming consumer is presumed to have a less incentive to hold equity due to its riskiness. Consequently, the habit-forming middle generation should have a stronger preference on bonds to equity than non-habit formation utility.

[5] Calibrating the equity premium defined as the difference between the return for equity and that of bonds in the OLG framework, Kim (2003) derives a higher equity premium than that obtained from the CDM framework by introducing the habit formation utility.

The rest of the paper is organized as follows. In section 2, we build up a model of habit formation utility in an overlapping-generations economy. The effect of habit formation utility on the middle-aged investors' savings is evaluated in section 3. Section 4 concludes the paper.

2. The Model

We consider a utility-maximizing representative consumer in an overlapping-generations economy, where each generation lives for three discrete periods as a member of the young, middle, and old generations. The representative consumer born at $t = 0$ with no endowment assets receives labor income w_0 in period $t = 0$, w_1 in period $t = 1$, and zero labor income in period $t = 2$. In the first period, the consumer receives a relatively low labor income sufficient only to satisfy his or her first period consumption. In the second period, the consumer receives wage income, and wants to accumulate assets for the third period consumption due to an expected zero labor income for this period. In the third and last period, the consumer retires and consumes the assets accumulated during the second period. Savings for smoothing lifetime consumption is done by holding a diversified portfolio with equity and bonds. However, following the setting by CDM (2002), we also introduce the borrowing constraint that prevents the young-aged generation from borrowing at the risk-free rate to hold equity, such that only the middle-aged generation is allowed to save with a diversified portfolio of equity and bonds. Therefore, over the life cycle each generation can be characterized as the borrowing-constrained young generation, the saving middle generation, or the dissaving old generation.

Following Sundaresan (1989) and Constantinides (1990),[6] we assume that the representative consumer's utility exhibits habit formation preferences, such that the habit level of consumption at time *t*, X_t, is a positive fraction of the consumer's own previous consumption level, *i.e.*, $X_t = \delta C_{t-1}$. δ is the constant habit persistence parameter and it is assumed to have a value between 0 and 1, which characterizes the consumption of non-durable goods and services.[7] Since the representative consumer in the first period does not have the previous period consumption for habit formation, the consumer is assumed to have a habit formation utility function from the second period on. Consequently, the consumer has the following sum of discounted utility flows over three periods:

$$U_0 = \frac{[C_0]^{1-\gamma}}{1-\gamma} + \beta \frac{[C_1 - X_1]^{1-\gamma}}{1-\gamma} + \beta^2 \frac{[C_2 - X_2]^{1-\gamma}}{1-\gamma}, \tag{1}$$

where C_0, C_1 and C_2 are consumption at $t = 0$, $t = 1$ and $t = 2$, respectively, and all are assumed to be positive.[8] Habit level at time *t* is determined by $X_t = \delta C_{t-1}$. β is the constant subjective discount factor, and $\gamma > 0$ is the constant risk averse parameter.

The representative consumer then faces the following budget constraints over life cycle:

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[6] Abel (1990) proposes another type of habit formation, *i.e.*, *Catching up with the Joneses*, where the habit-forming behavior is based upon the consumption of other consumers. Comparing his or her own consumption to that of others, a consumer could get utility from knowing that he or she is consuming more than others.

[7] The standard CRRA utility is a special case of habit formation utility with $\delta = 0$.

[8] To be specific, excess consumption over the habit, $C_t - X_t$, should be positive for each period's instantaneous utility to be defined.

$$C_0 \le w_0 - S_0, \tag{2}$$

$$C_1 \le w_1 + R_1 S_0 - S_1, \text{ and} \tag{3}$$

$$C_2 \le R_2 S_1 \tag{4}$$

where S_0 and S_1 are savings of young and middle generations, R_1 and R_2 are the gross rates of return in the middle and old periods, and w_0 and w_1 represent the labor income of young and middle generations. With Eq. (2), (3) and (4), the objective function U_0 becomes the following value function, V_0.

$$V_0 = \frac{[w_0 - S_0]^{1-\gamma}}{1-\gamma} + \beta \frac{[(w_1 + R_1 S_0 - S_1) - \delta(w_0 - S_0)]^{1-\gamma}}{1-\gamma} + \beta^2 \frac{[R_2 S_1 - \delta(w_1 + R_1 S_0 - S_1)]^{1-\gamma}}{1-\gamma} \tag{5}$$

It should be noted that the RRA coefficient of the habit formation utility is $\frac{\gamma \cdot W_t}{(C_t - X_t)}$, where wealth W_t is defined as $W_t = C_t + S_t$.[9] Since $W_t > C_t - X_t$, $\frac{\gamma \cdot W_t}{(C_t - X_t)}$ is greater than γ, which is the RRA coefficient of the CRRA utility. This implies that a habit-forming consumer has a lower (a higher) incentive to put more weight on equity (bonds) investment than a consumer with the CRRA utility.

[9] With $W_t = C_t + S_t$, $RRA = -W_t \cdot \frac{U''}{U'} = W_t \cdot \frac{\gamma (C_t - X_t)^{-\gamma-1}}{(C_t - X_t)^{-\gamma}} = \gamma \cdot \frac{W_t}{C_t - X_t}$.

3. Optimum Savings of the Habit-Forming Middle Generation

CDM (2002) assume that the young generation bears a restriction on borrowing against future labor income, which is realistic in that human capital alone does not collateralize major loans in modern economies for reasons of moral hazard and adverse selection. In addition to the constraint, the young-aged generation's labor income (w_0) is assumed to be at a lower level than w_1, so that it is enough only for the consumption at $t = 0$. These two assumptions together rationalize zero savings of the young-aged generation, *i.e.*, $S_0 = 0$. Since the young-aged generation is excluded from participating in the equity market, then the equity price (and, thus, the equity premium) is exclusively determined by the middle-aged (generation) investors' savings decision. Assuming only the CRRA utility in the overlapping generation economy, CDM (2002) show that imposing a borrowing constraint to the young-aged generation reduces the risk-free rate and increases the risk premium.

Compared with the CDM's framework as a benchmark model, the combination of the habit formation utility and an overlapping-generations economy yields better results that can be used for explaining the equity premium. The main idea is that (a) an introduction of habit formation preferences results in a stronger incentive to save in the middle period for the old period's consumption than the non-habit formation utility such as the CRRA utility suggested by CDM, (b) a habit-forming consumer will have a lower incentive to take equity investment to secure a stable consumption for the next period than a consumer with the CRRA utility. A habit-forming middle-aged generation, with a higher RRA coefficient, will put more weight on bond relative to equity for his/her diversified portfolio, thereby causing an even higher demand for bonds but less demand for equity.

Using comparative statics on the optimum savings decision of the middle-aged investors, we examine the effect of habit-formation preferences on the optimum savings level. First, we derive the optimum savings level of the middle-aged generation from solving the maximization problem of the discounted utility over the life cycle. Then, we show the positive impact of the habit formation preferences on the optimum savings level of the middle-aged.

Differentiating V_0 (with $S_0 = 0$) of Eq. (5) with respect to S_1 yields the first order condition *Q* as follows:

$$Q: \beta^2(R_2+\delta)[R_2S_1^* - \delta(w_1 - S_1^*)]^{-\gamma} - \beta[(w_1 - S_1^*) - \delta w_0]^{-\gamma} = 0, \tag{6}$$

where S_1^* is the optimal savings level of the middle-aged generation.[10] If the first and second order conditions are satisfied, the effect of habit formation on the optimum savings level of the middle-aged generation is determined by the sign of $\frac{dS_1^*}{d\delta}$, where $\frac{dS_1^*}{d\delta} = -\frac{\partial Q}{\partial \delta} \Big/ \frac{\partial Q}{\partial S_1^*}$. Thus, $\frac{dS_1^*}{d\delta}$ can be expressed as follows:

$$\frac{\beta^2[R_2S_1^* - \delta(w_1 - S_1^*)]^{-\gamma} + \beta^2\gamma(R_2+\delta)(w_1 - S_1^*)[R_2S_1^* - \delta(w_1 - S_1^*)]^{-\gamma-1} - \gamma\beta w_0[w_1 - S_1^* - \delta w_0]^{-\gamma-1}}{\gamma\beta[w_1 - S_1^* - \delta w_0]^{-\gamma-1} + \beta^2\gamma(R_2+\delta)^2[R_2S_1^* - \delta(w_1 - S_1^*)]^{-\gamma-1}} \tag{7}$$

Using the budget constraints, Eq. (7) can be rewritten as follows.

[10] The second order condition for the maximization problem is also satisfied as follows.

$-\gamma\beta[(w_1 - S_1^*) - \delta w_0]^{-\gamma-1} - \gamma\beta^2(R_2+\delta)^2[R_2S_1^* - \delta(w_1 - S_1^*)]^{-\gamma-1} < 0$.

$$\frac{dS_1^*}{d\delta} = \frac{\beta^2[C_2 - \delta C_1]^{-\gamma} + \beta^2\gamma(R_2 + \delta)C_1[C_2 - \delta C_1]^{-\gamma-1} - \gamma\beta C_0[C_1 - \delta C_0]^{-\gamma-1}}{\gamma\beta[C_1 - \delta C_0]^{-\gamma-1} + \beta^2\gamma(R_2 + \delta)^2[C_2 - \delta C_1]^{-\gamma-1}} \tag{8}$$

Since the implied result for the CDM's framework is that $\frac{dS_1^*}{d\delta} = 0$, $\frac{dS_1^*}{d\delta} > 0$ implies that habit formation has a positive impact on the optimum savings level, thereby showing a higher incentive to save more than the CDM's framework. We calibrate Eq. (8) to determine the sign of $\frac{dS_1^*}{d\delta}$ under a plausible range of parameters and the consumption set. Since it is always the case that $C_1 > \delta C_0$ and $C_2 > \delta C_1$ (in order to have positive utility levels), the denominator of Eq. (8) is positive. Thus, the sign of $\frac{dS_1^*}{d\delta}$ is determined by the sign of the numerator of Eq. (8).

Given permissible parameter values for β, δ, γ and R_2 under several combinations of consumption paths over the life cycle, calibrating the model readily confirms the following inequality:[11]

$$\beta^2[C_2 - \delta C_1]^{-\gamma} + \beta^2\gamma(R_2 + \delta)C_1[C_2 - \delta C_1]^{-\gamma-1} - \gamma\beta C_0[C_1 - \delta C_0]^{-\gamma-1} > 0. \tag{9}$$

[11] Boundaries of parameters and variables are determined as follows. For the value of discount factor, β, we assign around 0.955 per year. By recalculating β in terms of 20 years period (one generation period), we get 0.3982. For the habit persistence parameter, δ, we use the value 0.615 following Otrok, Ravikumar and Whiteman (2002). The value of γ is set between 1 and 10 following Mehra and Prescott (1985) and many other studies that utilize this range of values. For the value of consumption over different age cohorts, we use values between 20,000 and 40,000, which are consistent with the real Consumer Expenditure (conducted by the Bureau of Labor Statistics) Survey from 1984 to 1996.

The above inequality indicates that $\frac{dS_1^*}{d\delta} > 0$, which implies that habit formation preferences have a positive impact on the optimum savings level. The calibration result verifies that a habit-forming consumer saves in the middle period more than a nonhabit-forming consumer.

4. Conclusion

In this study, we extend Constantinides, Donaldson and Mehra's work (2002) by incorporating habit formation into an overlapping-generations economy. Using comparative statics and model calibrations, we have shown that incorporating habit formation preferences in the three-period OLG model has a positive impact on the savings level of the middle-aged generation. When compared to the CDM's case, where non-habit formation preferences are assumed, the explicit inclusion of habit formation within an overlapping-generations model results in a stronger incentive for agents to secure their future consumption, so that the habit-forming middle-aged investors will save even more than do the middle-aged investors in the CDM's case. This higher incentive to save causes a higher demand for bonds and less demand for equity, thereby yielding a lower risk-free rate and a higher required return for holding equity than does the CDM's framework. Therefore, accounting for habit formation preferences in the OLG framework can provide a more satisfactory explanation of the equity premium puzzle.

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