Private Tutoring As Rent Seeking and Effects of Some Measures to Curb It

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Abstract

Tuition control in the education market necessarily creates excess demand. The educational goods then must be rationed out through non-price competition. One non-price selection device commonly adopted is the admission test. Parents and students pour their resources in competition to win the prize, the admission ticket. In this paper, we build a simple choice-theoretic model to analyze how much would be spent for this kind of rent-seeking activity and how the aggregate amount would respond to various measures to discourage it. We show that many seemingly plausible measures will not generate the desired effects.

JEL classification: I2, D72

Key words: private tutoring, rent-seeking, rationing, contest

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1. Introduction

For various reasons, tuition controls are practiced in many higher educational markets, both in advanced as well as underdeveloped countries, with one conspicuous reason being making the opportunity for higher education available to as many students as possible, especially those from poor families. Whatever the reasons might be, however, the economic effects of the tuition control are similar almost everywhere and every time.

First, a large part of the actual benefit does not accrue to poor families but to middle or high income families because a larger portion of college students are in fact coming from relatively rich families. Second, it almost always creates an excess demand in those markets. The number of students who are seeking opportunities for advanced study is in excess of the supply where prices are regulated below the market clearing level. The school authority, constrained not to raise the tuition to clear the market, therefore, has to adopt a certain kind of rationing device in allocating the limited slots to a large number of contending students. Administering various forms of admission tests is the most conventional way of selecting the students[1]. Students then compete to get higher marks in the tests, and thereby increase their chances of getting admitted.

Obtaining an admission in the situation is just like winning a prize in a contest. If one student succeeds in getting in, he can enjoy a windfall gain and so he becomes a prize winner. If he fails, he is just a loser in the contest. The prize, the gap between his demand price and the controled price, therefore, drives students and their parents to compete in various ways; they may study or practice harder at home, attend private educational institutions, hire private tutors. Some may even attempt illegal activities such as forging application documents, bribing the person in charge to get tips on test questions or fabricate the results. In short, they will do whatever will help to raise the chances of their being admitted.

In a way, self-study and/or private tutoring are complementary to school

1) Even in the absence of excess demand, schools may still administer tests to sort out different student abilities. Because of the peer effects, securing a relatively homogeneous group as a single educating unit is more effective and less costly. Rothschild and White (1995) and Stiglitz and Weiss(1981).

education, and are sometimes needed for their own. But in many countries, the highly selective admission procedure induced by the tuition control and resulting competitive frenzy are creating various social problems with them; financial burdens for the families are substantial and the stress imposed on the students are by no means trivial. In Korea, the issue of "excessive" private tutoring has been chronic social headache since early 1960s, and the government has tried various policy measures to abate the pressures[2]. Japan, with its exceptionally tight educational system and fierce competition among students has also been known to have grappled with similar problems for many years. France has its own long history in this matters, and modern China is not an exception[3]. According to a recent report (Bray, 1999), private tutoring is prevalent in many more countries, such as Sri Lanka, Malta, Egypt, Thailand, Mauritius, Hong Kong, Morocco, Tanzania, Zimbabwe and Taiwan.

The main purpose of this paper is to build a simple model in which the above mentioned private tutoring activities in educational markets can be examined. The private tutoring is seen in this paper as one of the rent seeking activities in the educational market under tuition control. Parents and students compete for grabbing the rent artificially created by the tuition control.

Existing literature abounds which deals with rent seeking activities of this kind; contest models in a public choice theory and tournament models in labor economics are only a few among many[4]. What is newly attempted in the paper is to develop a model with a continuum of players in the hope that it may generate richer implications and a handy tool for comparative statics analysis. With few exceptions, most existing literatures in this area limit themselves to the cases where there are only two contestants. And because of this modelling rigidity, some interesting issues are not fully developed.

In a pair-wise competition model, if one wins, the other necessarily loses. In a model of continuum of players, the number of winners and losers can be

2) See Adams and Gottlieb(1993), Park(2000), and Korean Ministry of Education(1998).

3) For some illustrative stories for these countries, see Little and Wolf(1996), Dore(1997).

4) For contest and tournament models, see Lazear and Rosen(1981), Nalebuff and Stiglitz (1983), McLaughlin(1988), and Nitzan(1994). In many cases, the tournament models are not directly concerned with rent seeking activities. They are more concerned with the problem of eliciting optimum amount of productive efforts by setting up an incentive payment system. A slight modification and reinterpretation, however, will allow it to be transformed into a contest model most of which directly aims to explain rent seeking activities.

freely controlled. This fits more squarely with the situation described above where a large number of contestants compete to get the coveted items and multiple winners are selected[5].

Selecting a small number of winners out of a large number of contestants requires a kind of formal or informal criterion according to which the selection is made. Selection criterion can be either a standard or a quota. In a standard system, the ones whose qualification are determined to lie above the pre-set value will be chosen, whereas in a quota system, those whose relative ranks are equal to or higher than the pre-set number will be chosen. Depending on which system is used, the incentives faced by the contestants become different. Continuous agent model allows us to develop a model which can consistently handle these two distinctive selection criteria in a coherent way.

Since private tutoring as a rent seeking is inherently unproductive and wasteful from social standpoint, governments in many countries have implemented various policy measures to stem or reduce it, ranging from an outright ban of private tutoring (in China and Korea) to designing a new, more elaborate test format that will help select qualified students while allaying the cost of tutoring. We examine the effectiveness of some of those anti-private-tutoring measures.

One noble finding is that the effectiveness of policy measures to suppress the rent seeking is in many cases indeterminate, and crucially depends on the current degree of competitiveness in the market. For example, if the current competition to get in college is highly competitive, making it easier to get admission either by lowering the cut-off score or increasing the quota does *not* reduce the aggregate amount of private tutoring. Instead, one should make it even harder to discourage private tutoring. In short, under quite general and plausible environments, there is no simple linear relationship between the degree of competitiveness and the aggregate amount of rent seeking, which is somewhat counter -intuitive on first impression.

Devising a test procedure which will render the private tutoring to be less reliable means to improve the score is not a good way to curb the private

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5) A tournament model with multiple contestants has been developed formally in Green and Stokey(1983) and touched briefly in Nalebuff and Stiglitz(1983) and O'keeffe, Viscusi and Zeckhauser(1984). All are, however, mainly concerned with the optimum payment scheme rather than rent seeking.

tutoring, either. Increasing the random elements in the test will have differential impact on students' chances of winning, depending on where their expected scores are located. As a result, the average quality of students admitted under this system will deteriorate. Furthermore, the total amount of private tutoring aggregated over the whole population varies differently, depending again on the current state of competitiveness in the college admission.

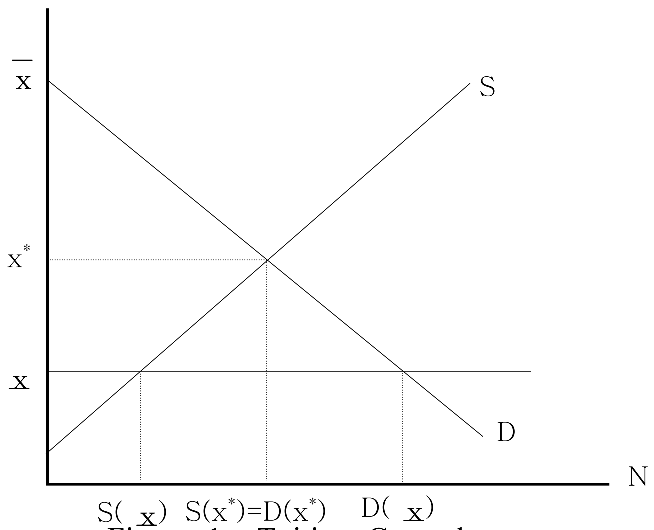
Under the quota system, things get further complicated because a fixed quota necessarily introduces negative externalities among students. At the margin, one student's admission necessitates one other student's rejection. When there is a policy shift, therefore, some students will gain, and others will get hurt. In certain cases, schools may end up with undesirable outcomes, in the sense that students with stronger aspiration for study, and thus, higher value on education may suffer from diminished odds for admission, while students with lower value enjoy increased chances.

Although the paper is mainly concerned with the private tutoring problem in the educational market, the model developed here can be applied, with little change, to a great variety of markets under price control. The divergence between the demand price and the regulated price is unavoidable consequence of the price control of any type. Thus, we face exploitable rents in these markets. Competition to get the goods in question is then just a rent seeking. The selection process may not be as formal as in education market, but there still exist various forms of selection criterion, implicit or explicit, and people will make investment in those activities which they conceive will enhance their chances of meeting the criterion. And various anti-rent-seeking measures may be introduced in those markets.

The organization of the paper is as follows. In the next section, I set up a basic model in which a private tutoring as a rent seeking is analyzed in a setting of a pre-set quality standard. The effects of various policy measures to suppress the private tutoring are discussed. In Section 3, the model is expanded to cover the cases where admission quota is fixed. The effects of policy measures are again taken up in this expanded setting. Conclusions are in Section 5.

2. Basic Model

Let's consider a college education market. The demand is given as D(X), and the supply as S(X), where X stands for the tuition that a student has to pay. As usual, the demand schedule is assumed to be downward sloping and the supply schedule upward sloping, $D'(X)<0$ and $S'(X)>0$. Let's assume that each student wants to purchase just one unit of education. Thus, the horizontal axis measures the number of students who want to purchase the educational service from colleges. The value that each student attaches to college education is given by X, the height from the horizontal axis to the demand schedule. The maximum value, the intercept of demand curve with the vertical axis, is assumed to be $\bar{x}$.



<Figure 1> Tuition Control

If there is no regulation on tuition in the college education market, the equilibrium tuition will be determined at x^*, and the market will be cleared. If the government imposes a ceiling on the level of tuition below x^*, the market will be in disequilibrium. Let's assume the tuition ceiling is set at $\underline{x}$ $(<x^*)$. Students whose personal valuation on education is greater than $\underline{x}$ will want to get in the college and are willing to spend on something which can help increase his chance for getting admitted to college. Once he is admitted to

school, the surplus he can enjoy is, therefore, given by $X-\underline{x}$. The total number of students who is willing to participate this selection contest is given $D(\underline{x})$ which is obviously greater than $D(x^*)$. Under the regulated scheme, the supply of educational service is constrained to be $S(\underline{x})$ ($<S(x^*)$). Then, the admission rate, which is the inverse of the competition rate, is given by $\phi(\underline{x}) = S(\underline{x})/D(\underline{x})$.

I start with a simplest model, and then will try to zero in step by step by adding more constraints until it would appear to fit the general picture described above[6]. Let's start with a simple contest model in which the contestants have varying valuations for the objects that they are competing for[7]. It is assumed that the valuations of the prize, X, ($X \geq 0$) which is distributed in the range of [$\underline{x}$, $\bar{x}$], has the cumulative density function, H(X).

There will be an admission test to select the qualified students. Students will put forth effort, denoted as e, to obtain high scores in the test. The effort can be anything which can improve his odds of winning the prize, so it will depend on the characteristics of the test adopted. More specifically, the effort in this paper is interpreted to represent the amount of private tutoring a student purchases outside of regular schools. It is assumed that the tutoring has no intrinsic value or productivity in itself other than improving his score in the admission test. In this regard, the student can be said to be engaged in rent seeking, the rent associated with the college admission. The student will spend valuable resources to grab the rent as much as he deem worthwhile.

The test score will be distributed as

$$T = \alpha e + \varepsilon, \quad \alpha > 0. \tag{1}$$

6) The model developed below is quite similar to Kim(2000) although there are several differences. Kim considers the problem of optimum test design assuming that students have different intrinsic ability. His main concern was to design a test structure which can select the most able people to assign to two different tasks so that total social welfare is maximized. He doesn't relate his model with the issue of tuition control. In this paper, in contrast, the contestants differ not in their ability but in their valuations of the prize they will get when they become winners. And the individual gains are directly affected by tuition control. Furthermore, the tuition control has also an effect on the number of people who are willing to participate in the selection contest, and on the number of people who will be selected for the prize.

7) According to O'keeffe, Viscusi and Zeckhauser(1984)'s terminology, the model described here is an "uneven" but "fair" contest.

The noise term, ε, has a mean 0, and variance σ^2, and has density function f, and cumulative distribution function F. It is assumed that f is symmetric around 0, and $f'>0$ when $\varepsilon<0$ and $f'<0$ when $\varepsilon>0$. Given e, an individual student's expected test score is αe, i.e., $E(T)=\alpha e$.

Taking effort or purchasing private tutoring is costly, and the cost function is given by $\beta c(e)$, $\beta>0$, where β is a parameter affecting the marginal cost of private tutoring. About the general shape of the cost function, it is assumed that

$$c'>0, \quad c''>0, \quad c'(0)=0. \qquad (2)$$

Students whose score is greater than the cut-off level, t^*, will be admitted. Later, this cut-off score will be determined endogenously in such a way that it would be compatible with a given number of students who will be admitted. In the meantime, the cut-off score is assumed to be fixed. It should be noted at this juncture, however, that the fixity of the cut-off score is just a matter of degree. Even when the total number of admissions is strictly controlled by the relevant authority, schools occasionally apply a kind of implicit quality standard to maintain the overall quality of the students, sometimes even risking the possibility of having unoccupied slots in the school, especially when the overall quality of the applicants turns out to be poor. In this sense, the assumption of fixed cut-off score has still some practical relevance.

Given the cut-off level of score, t^*, the probability of being admitted is given as

$$\begin{aligned} Prob(T>t^*) &= Prob\ (\alpha e + \varepsilon > t^*) \\ &= Prob\ (\varepsilon > t^* - \alpha e) \\ &= 1 - F(t^* - \alpha e). \qquad (3) \\ &= F(\alpha e - t^*) \end{aligned}$$

The last equality comes from the assumption of symmetry of F which assures the relationship, $1-F(z)=F(-z)$. This transformation allows us more intuitive and straightforward interpretation of some of the results below. Under this transformation, F has the meaning of probability of success which is increasing in effort level, e, and f the marginal probability of success.

The student's optimization problem is then

$$\max_{e} \quad X\, F(\alpha e - t^{*}) \; - \; \beta c(e). \tag{4}$$

The FOC is

$$\begin{aligned} &X\, f(\alpha e - t^{*})\, \alpha \; - \; \beta c'(e) \; = \; 0 \qquad \text{for} \quad e > 0, \\ &X\, f(\alpha e - t^{*})\, \alpha \; - \; \beta c'(e) \; \leq \; 0 \qquad \text{for} \quad e = 0. \end{aligned} \tag{5}$$

The FOC simply says that the marginal gain from an additional tutoring must be equated with its marginal cost for interior solution. Otherwise, the equilibrium effort will be set at 0.

The corresponding SOC is

$$X\, f'(\alpha e - t^{*})\, \alpha^{2} \; - \; \beta c''(e) \quad = \quad \mu \quad < \; 0. \tag{6}$$

Since $f'>0$ when $\varepsilon<0$, the SOC may not hold with an arbitrary density function, especially when the distribution is highly concentrated and/or the marginal cost function is quite flat. In the following, it is assumed that the variance of the distribution, σ^2, is sufficiently large, and/or the marginal cost function is sufficiently steep so that the SOC is always satisfied[8].

From the FOC, we can solve for the optimum amount of private efforts as a function of parameters of the model,

$$e^{*} = e^{*}(X,\ t^{*},\ \alpha,\ \beta,\ \sigma), \tag{7}$$

and perform the comparative statics with respect to each parameter. Since we are assuming that private tutoring is a unproductive rent seeking activity, we will

8) The possibility that the SOC condition may not be met appears in most of the tournament models. See Lazear and Rosen(1981), Nalebuff and Stiglitz(1983) and McLaughlin(1988).

use the following social welfare function which should be minimized as much as possibleies in evaluating the efficacy of various policy measures to curb the private tutoring.

$$W(t^*, \alpha, \beta, \sigma) = \int_{\underline{x}}^{\bar{x}} \beta\, c[e^*(t^*, \alpha, \beta, \sigma)]\, dH(X). \tag{8}$$

The Effects of Varying Valuations Among Individuals

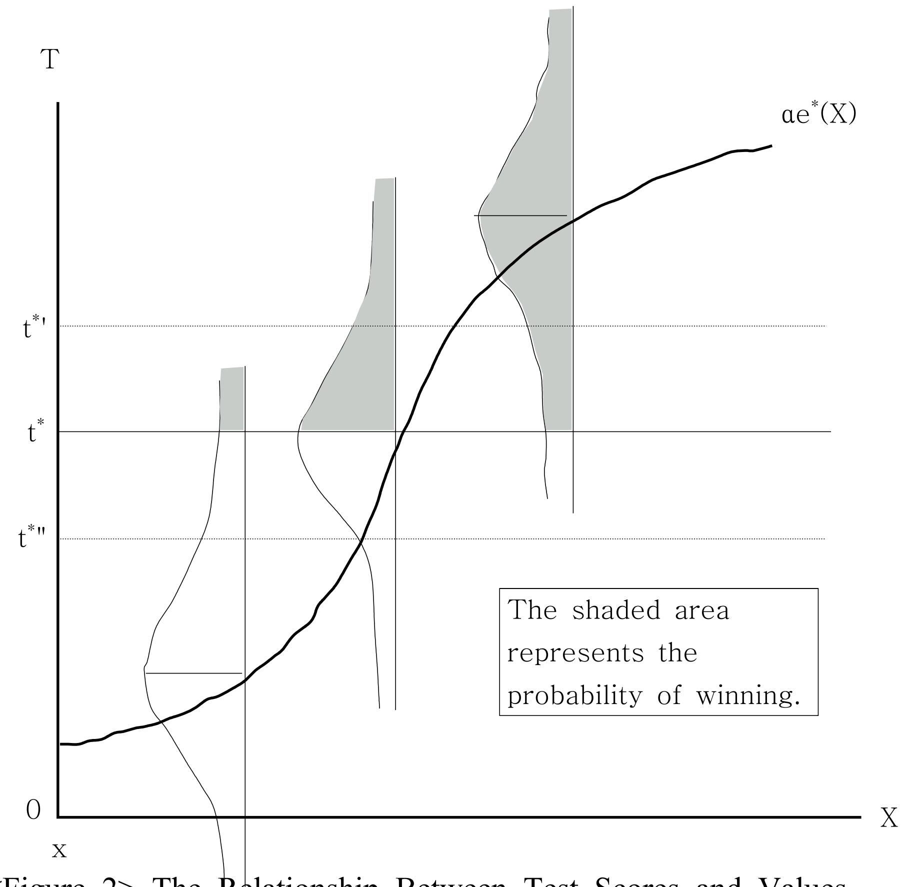
Differentiating the FOC with respect to X and rearranging the terms yields,

$$\frac{\partial e^*}{\partial X} = \frac{-\, f \alpha}{\mu} > 0. \tag{9}$$

The denominator is negative from the SOC, and since f>0 everywhere, the whole expression is positive. In other words, a student who puts a higher value on education will purchase more tutoring to improve the test score [9]. If c'(0)>0, some students who are located near the lower end of the distribution, $\underline{x}$, may choose 0 effort and thus drop out of the contest. In order to ensure that every student with a positive surplus value participate in the game and exert a certain positive amount of effort, I have imposed the constraint, c'(0)=0, in (2) above.

One interesting thing is that for a student with a given X, his equilibrium effort level is at its highest when he is near the cut-off score. This comes from the fact that f is symmetric around 0 and has the largest value at ε= 0. The student whose expected score is far below the cut-off point does not work hard because his marginal gain from the incremental effort is expected to be small. By the similar logic, the student whose expected score is way above the cut-off point does not work hard, either, because his chance of being admitted is not affected that much by his additional effort.

9) This accords with the general results in tournament models that the equilibrium effort is increasing in the prize spread between the winning and losing prize. In the current model, the loser gets 0 payoff.



<Figure 2> The Relationship Between Test Scores and Values

A diagram is helpful for understanding the relationship explained above. Note that $f'>0$ when $\varepsilon<0$, and $f'<0$ when $\varepsilon>0$. Consequently, the value of $f(\alpha e^{*}-t^{*})$ increases monotonically when $\alpha e^{*}(X)<t^{*}$, and reaches peak at $\alpha e^{*}(X)=t^{*}$, and then decreases when $\alpha e^{*}(X)>t^{*}$. The slope of the function $e^{*}=e^{*}(X)$ should, therefore, increase up until αe^{*} reaches t^{*} and then decrease beyond that point, yielding a S shaped curve like in <Figure 2>. Note also that as t^{*} moves upward (to t^{*}' in the <Figure 2>) , more students fall into the range in which each student's f' at his $\alpha e^{*}-t^{*}$ takes a positive value. If t^{*} moves downward (to t^{*}" in the <Figure 2>), the opposite result occurs.

With the result, it is easy to show that the student with a higher value on education will have a higher probability of being selected. By differentiating $F(\alpha e^{*}-t^{*})$, the winning odds, with respect to X, we obtain the following.

$$\frac{\partial F}{\partial X} = f\alpha \frac{\partial e^*}{\partial X} = -\frac{f^2\alpha^2}{\mu} > 0. \tag{10}$$

The fact that the student who has a higher value on college education will exert more effort guarantees the whole expression above to be positive, implying students with greater values on education will have higher probability of getting admitted. The selection process is, therefore, conducive to social efficiency in the sense that the opportunities for higher education are allocated to those who attach relatively higher values on education.

Changing the Cut-Off Score of the Test

The effect of changing the cut-off level of the test score on the effort level can be obtained in a similar way. By differentiating the FOC with respect to t^*, we have

$$\frac{\partial e^*}{\partial t^*} = \frac{Xf'\alpha}{\mu}. \tag{11}$$

The direction of change in the above expression depends on the sign of f'. It is positive when $f'<0$, that is, when $\alpha e^*(X)>t^*$. The students whose expected scores are higher than the cut-off point will work harder because their stakes are now threatened by the increased cut-off point, and their marginal returns to efforts are made higher. The direction of change is negative when $f'>0$, in other words, when $\alpha e^*(X)<t^*$. It implies that the students whose expected test scores are below the cut-off point will decrease their effort level in response to the increased cut-off point. They will take more relaxed attitude because their chances of being admitted have gone farther away by the upward movement of the cut-off point. For them, additional efforts are no longer worth as much as they used to be. (see <Figure 2>.)

Even though each student is making adjustment in different directions depending on his own relative location in the distribution, the odds of success will invariably decrease for every student by the hike of cut-off point. Differentiation of $F(\alpha e^{*}-t^{*})$ with respect to t^{*} yields [10],

$$\begin{aligned} \frac{dF(\alpha e^{*}-t^{*})}{dt^{*}} &= f\left(\alpha \frac{\partial e^{*}}{\partial t^{*}} - 1\right) \\ &= \frac{f\beta c''}{\mu} < 0. \end{aligned} \tag{12}$$

For an obvious reason, the student who has chosen to work less in response to the increased cut-off point will have a smaller chance of being admitted. The student who has chosen to work harder will not be able to improve his/her odds, either, because he/she cannot fully recover the loss of odds due to the cut-off increase by working harder. Surely, as he/she works harder, his/her score will improve, but not as much as the whole amount of the change in the cut-off point. As a result, part of the gap between his/her expected score and the cut-off point, $\alpha e^{*}-t^{*}$, will remain shrunk. This will lower his/her winning probability, F, too. The reason why he/she cannot fully recover the gap between his/her expected score and the cut-off point is that he/she has already taken large enough effort (Note that he/she is the one whose expected score exceeds the cut-off score.) and increasing his/her effort further entails a cumulative cost increase.

One important implication of the above result is that the total number of students who will be admitted will invariably decrease when the cut-off point is raised, and will increase when the cut-off score is lowered, regardless whether students work harder or not following the hike or cut.

There is one more implication which is equally important. In aggregate, if the test is highly competitive in the sense that the cut-off score is set at so high a level that only those students with very high X's can pass, most students will fall in the range of $f'>0$[11]. In this case, making the examination more difficult

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10) We use the notation dF/dt^{*} to distinguish it from the direct effect, $\partial F/\partial t^{*}$, which is just $-f$, even though there are still some other parameters held constant. In the following, the same rule about the notation on derivatives will be applied; whenever the effect of a parameter change is evaluated in such a way that it contains not only the direct effect but also the indirect effect via the endogenous variables of the model, it is denoted as total derivatives, despite the fact there remain some fixed parameters. Needless to say, it is a bit confusing. But there appears to be no other good alternatives since we do not have yet a good notation for partial total derivatives.

by raising the cut-off score will have an effect of discouraging a larger number of students and making all of them work less. (see <Figure 2>). On the other hand, making an easy test a bit more difficult-- easy in the sense that the cut-off point is set at a low level so that a large number of student can pass the exam -- will make a large number of students, who used to be above the cut-off line, become alert and push them to work harder.

In short, making a hard test harder will make many students discouraged and work less. Whereas, making an easy test harder will induce many students to work more. This implies that if the policy objective is to minimize the rent seeking activities, it would be desirable to make a hard test harder, or an easy test easier because it can thereby reduce the aggregate amount of rent seeking activity of the whole population. Making a hard test easier or an easy test harder will aggravate the situation by increasing the aggregate volume of rent seeking expenditures.

Still in other words, an effective way of reducing the aggregate expenditures on rent seeking is to push the cut-off point toward either the upper or lower extreme end. It will make most of the students either discouraged or relaxed, and thereby the additional efforts unworthy. On the other hand, moving the cut-off score toward the mid-point will make most of the people either newly alerted or revived, and the additional rent seeking will appear more attractive.

More formally, differentiating the social objective function (8) with respect to t^* gives,

$$\frac{dW}{dt^*} = \int_{x}^{\bar{x}} \beta c' \frac{\partial e^*}{\partial t^*} dH(X). \tag{13}$$

Therefore, the sign depends on the sign of $\partial e^*/\partial t^*$, which in turn depends on the location of t*. If it is located near the upper end of the score distribution, then in a wide of range of X, the partial derivative inside of the integral will take negative value, rendering the sign of the whole expression to be negative.

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11) In the extreme, if we set the cut-off score at a very high level, say at $\alpha e^*(\bar{x})$ so that the student with the highest value, $\bar{x}$, can have 50% chance of winning, then everybody else will have f'>0.

If the cut-off point is located near the lower end, the opposite conclusion would ensue.[12].

Reducing the Effectiveness of the Private Tutoring on Test Scores

Another measure that may be contemplated by the public authority which intends to discourage the private tutoring is to design the test format in such a way that private tutoring becomes less effective in raising the test scores[13]. The effect can be analyzed simply making the size of α smaller in our model.

Differentiating the FOC with respect to α and rearranging gives,

$$\frac{\partial e^*}{\partial \alpha} = \frac{-Xf}{\mu} + \frac{-Xf'\alpha e^*}{\mu}. \qquad (14)$$

The first term is positive and the sign of the second term depends on the sign of f'. The first term captures the pure effect of increasing the marginal product to effort, α, on the marginal revenue, holding the marginal probability of winning constant. One unit increase in α pushes up the marginal revenue by Xf. This enhanced marginal returns to effort then induces everybody to increase his effort.

The second term captures the effects of an increase in α that works through the change in marginal probability of winning. As the expected score gets higher due to an increase in α, the marginal probability of winning is

12) For example, in Korea, the college entrance examination has been highly competitive, with the competition ratio standing somewhere around 3:1 to 4:1. In this situation, making the exam easy is likely to bring forth the opposite effects to what the Korean government is intending for. That is, it will further encourage the private tutoring activities among students, especially those who otherwise would have given up in some sense.

With a given number of college applicants, making the test easier by lowering the cut-off score necessarily results in an increase in the total number of college admissions. In this regard, the above result implies that increasing the college admissions will not have an effect of reducing the private tutoring as long as the current competition rate remains higher than 2:1.

13) A high ranking Korean government official was reported to have made the remark that, in the coming years, the test will be designed in such a way that having private tutoring will turn out to be totally worthless.

affected differently depending on where he is located. For a student whose expected score is below the cut-off line, $\alpha e^*(X)<t^*$, a higher score moves him closer to the cut-off line, and it raises his marginal probability of winning. Consequently, the student works harder. In the above expression, it means $f'>0$, which makes the second term positive, thereby rendering the whole expression to be positive. In short, those students whose expected scores fall short of the cut-off point (who put relatively lower value on college education) will study harder as the returns to study get higher.

Those students whose expected scores are already higher than the cut-off line, $\alpha e^*(X)>t^*$, however, work less as far as the effect of the second term is concerned when the marginal returns to effort gets higher. For them, a higher expected score moves their whole test score distribution upward, pushing further away from the cut-off point and this makes the marginal gain in their success probability smaller, that is, $f'<0$. As a consequence, the second term becomes negative for them.

The overall effect is then the sum of these two effects. Since it is impossible to proceed further with an arbitrary density function, I will assume a normal distribution and show that those who have extremely high expected score will in fact reduce their effort. With a normal density and letting $y = \alpha e^*-t^*$, we have

$$\begin{aligned} f'(y) &= \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{y^2}{2\sigma^2}\right] \left(-\frac{y}{\sigma^2}\right) \\ &= \left(-\frac{y}{\sigma^2}\right) f \end{aligned} \tag{15}$$

The combined effect will be negative only when $-Xf - Xf'\alpha e^* = -X\,[f + f'\alpha e^*] > 0$, alternatively, when $f + f'\alpha e^* < 0$. Substituting and solving gives

$$f + f'\alpha e^* = f\left[\,1 - \frac{y\alpha e^*}{\sigma^2}\right] < 0. \tag{16}$$

Solving this for αe^*, we obtain,

$$\alpha e^{*} > \frac{t^{*}}{2} + \frac{\sqrt{t^{*2}+\sigma^{2}}}{2}. \qquad (17)$$

Those students whose expected scores are higher than the value in the right hand side of the above inequality, therefore, work less when the returns to private tutoring get's higher.

Although some students with high enough scores choose to reduce their effort level, their odds to win the prize do not decrease. In fact, every student's odds, regardless of his/her initial score, will improve as a result of this increased returns to efforts. Differentiating $F(\alpha e^{*}-t^{*})$ with respect to α, we have

$$\begin{aligned}\frac{dF}{d\alpha} &= f\left(e^{*} + \alpha\frac{\partial e^{*}}{\partial \alpha}\right) \\ &= \frac{-f\beta(c''e^{*}+c')}{\mu} > 0.\end{aligned} \qquad (18)$$

Thus the odds improves for every student. The reason why the odds improves even for those students whose effort levels were lowered is that the increased marginal returns to per unit of effort outweigh the reduced effort effects.

The above results imply that an examination test which would cripple the marginal returns to private tutoring would be in general effective in reducing the expenditures on private tutoring.

Increasing the Noise of the Test

Still another conceivable measure to abate the incentive for the private tutoring might be to construct a test such that the test score should be relatively unpredictable. If private tutoring is unreliable means to improve the test scores, people will, on first impression, make less investment in tutoring. But this intuition does not bear scrutiny.

Upon differentiation of the FOC with respect to σ, we obtain the following equation,

$$\frac{\partial e^*}{\partial \sigma} = \frac{-\alpha X f_\sigma}{\mu}, \qquad \text{where} \qquad f_\sigma = \frac{\partial f}{\partial \sigma}. \tag{19}$$

Therefore, the overall sign depends on the sign of f_σ. Again by resorting to the assumption of normality, we have,

$$f_\sigma(y) = \frac{y^2 - \sigma^2}{\sigma^3} f(y). \tag{20}$$

Plugging it into (19) yields,

$$\frac{\partial e^*}{\partial \sigma} = \frac{-\alpha X f}{\mu} \left(\frac{y^2 - \sigma^2}{\sigma^3}\right), \tag{21}$$

implying that the sign of the change depends on the relative size of y^2 and σ^2; it is negative in the range of $t^*-\sigma < \alpha e^* < t^*+ \sigma$, and positive outside of the range. This suggests that the students whose expected scores are in the middle range around the cut-off point will decrease their efforts whereas the students who are located near the both tails of the distribution will increase their efforts.

This result is quite intuitive if we look at the changes made in the shape of the density function by the increased dispersion. Augmenting the variance decreases the density in the middle range and makes the tails of both ends fatter. Or, to put the same story in a slightly different way, the students in the upper tail work harder to guard against the possibility that their chances will be marred by the increased randomness of the test outcomes. The students in the lower tail also work harder but for a slightly different reason. They get into the bet in an anticipation that they may get admitted, this time just by sheer chance.

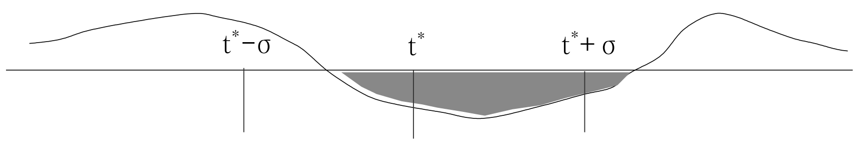
The winning odds for each student moves in a bit complicated way. For some students, the odds improve as a result of the increased dispersion, and for some others, they deteriorate. And still for some others, the signs of changes are indeterminate. To see this, let us take a differentiation of F with respect to σ. The result is,

$$\frac{dF}{d\sigma} \Big|_{t^*=exog} = F_\sigma + f\alpha \frac{\partial e^*}{\partial \sigma} \tag{22}$$

where $F_{\sigma} = \partial F / \partial \sigma$. The first term captures the pure effect of the σ change on winning odds, holding the effort level constant, and the second term the effect of the effort level change due to the increased noises in the test.

Let's take the second term first. It is easy to interpret because it contains the expression, $\partial e^{*} / \partial \sigma$, whose sign is indeterminate as seen above. The sign of the first term also depends on the location of student's expected score, but this time relative to the cut-off point itself, t^{*}. For any density function which has a unimodal and symmetric around the zero mean, as assumed at the outset, $F_{\sigma} > 0$ when $\varepsilon<0$, and $F_{\sigma} < 0$ when $\varepsilon>0$. In the current setting, therefore, the first term is positive for those students whose expected score is below the cut-off score, and negative for those students whose score is above the cut-off score.

Combining these two effects to get the overall picture about the effects on the winning odds of increased noises gives the following diagram, <Figure 3> below. The dark colored range represents where the winning odd effects are negative, and the white positive. As shown in the diagram, the odds improves for those students who are located near both the lower and higher ends of the distribution, and those in the middle range suffer. Since the demarcation lines are skewed to the right of t^{*}, the average quality of the student body will deteriorate as a result of the increased variance of the test.



<Figure 3> Overall Effect of Variance Change on Winning Odds

Increasing the Cost of Private Tutoring

Finally, still another measure that can be employed for the purpose of reducing the private tutoring is to make private tutoring more costly to take [14]. In the present model, increasing the cost means to raise the value of β.

Differentiation of the FOC with respect to β renders the following expression.

$$\frac{\partial e^*}{\partial \beta} = \frac{c'}{\mu} < 0, \tag{23}$$

which invariably insures that the efforts will decrease in response to the increased cost. Whether this will reduce the aggregate social cost of private tutoring, however, is uncertain because the per-unit marginal cost of tutoring has been increased even though the total amount of private tutoring is clearly decreased. Differentiating the social objective function, (8) with respect to β, gives,

$$\frac{dW}{d\beta} = \int_{x}^{\bar{x}} [c + \beta c' \frac{\partial e^*}{\partial \beta}] \, dH(X). \tag{24}$$

The second term inside the integral is negative from the above, but the first term is positive, and therefore, the net result is indeterminate [15].

As every student cuts down his/her effort level in response to the increased cost, the winning odds decreases for every student, for a trivial reason.

$$\frac{dF}{d\beta} = f\alpha \frac{\partial e^*}{\partial \beta} = \frac{f\alpha c'}{\mu} < 0. \tag{25}$$

3. Rent Seeking Under Quota System

So far, I have assumed that the cut-off score, t*, is exogenously given. So the model described above pertains to the cases where there are some kinds of

14) Legally banning the private tutoring altogether is certainly one option, which can be interpreted as making the cost prohibitively high.

15) Apparently, it depends on the elasticity of the total cost with respect to β. Rewriting the inside of the integral of equation (25) gives $c[1+(\beta/c)(dc/d\beta)] = c[1+\eta_{c\beta}]$ where $\eta_{c\beta}$ is the elasticity of the cost with respect to β.

pre-set standards for passing. There is a variety of tests which fit this category; most certificates or licenses are issued only those who pass the minimum quality standard, and some government administrative and judiciary officials are selected in this way.

There are, however, other tests where the total number of winners, not the cut-off score, is pre-set. In this system, therefore, whatever the distribution of scores may be, the selection criterion is not the absolute individual score but his relative ranking, and those people whose rank is above the pre-set number will be selected. We can call the former as the "Quality Standard System" and the latter as the "Quota System". In most cases, student selection for admission is administered by the quota system described above, the more so in the case that the total number of admission tickets is controlled by education authority, as is done in many underdeveloped countries. In the following I try to incorporate this new mode of selection method into the model and analyze its implications.

Basically, the model is the same as before except that we need one additional constraint that assures that the cut-off score now has to be set at the level that would be compatible with the pre-set quota. The system works like the following. Initially, ex ante, students form expectations about the cut-off score that will be chosen by the test authority given the fixed number restriction, n. And, on the basis of this expected cut-off score, they choose their optimum level of efforts, i.e. the amount of their tutoring purchase. Then the ex post cue-off score will be determined by the school authority at the level of the n^{th} highest score. In the final equilibrium, students' expectations are realized in the sense that the ex ante expected cut-off score is equal to the ex post cut-off level chosen by the school authority.

More formally, from the FOC and equation (7), we can see that for a student with value X, the probability that his score falls short of an arbitrary level of score, t, is given as

$$\begin{aligned} \text{Prob}\,[\alpha e^*(t*, X, \ldots) + \varepsilon < t] &= \text{Prob}\,[\varepsilon < t - \alpha e^*(t*, X, \ldots)] \\ &= F[t - \alpha e^*(t^*, X, \ldots)] \qquad (26) \\ &= 1 - F[\alpha e^*(t^*, X, \ldots) - t]. \end{aligned}$$

By taking the weighted average of this individual distribution of test score over

X, we can get ex ante population distribution of test score, G(t).

$$G(t) = 1 - \int_{\underline{x}}^{\overline{x}} F[\alpha e^*(t^*, X, \ldots) - t]\, dH(X). \tag{27}$$

In equilibrium, the fraction with t greater than t^* in G(t) must be equal to the admission rate, ϕ, where ϕ=n/N, with N denoting the total number of applicants [16]. Alternatively, 1-G(t^*)= ϕ or simply, $\phi = \int F\, dH(X)$ must hold.

Under this reformulation, the whole system is now represented by the following two equations with two endogenous variables, e^* and t^*.

$$\begin{aligned} &1)\quad X\, f[\alpha e^* - t^*]\, \alpha - \beta c'(e^*) = 0, \\ &2)\quad \phi = \int_{\underline{x}}^{\overline{x}} F[\alpha e^* - t^*]\, dH(X). \end{aligned} \tag{28}$$

The equation 1) is just the replication of the FOC given in (5) in the previous section[17]. The equation 2) is the rational expectation perfect foresight condition.

Now that we have set the stage for this expanded model, we can proceed to examine some of the effects of policy changes as we have done in the previous section.

Increasing the Admission Ratio

16) In a strict sense, fixed number system is different from fixed ratio system, especially when the total number of applicants varies every time. In this section, I am assuming the total number of applicants is fixed. Therefore, both are the same.

17) This is not innocuous, though. In a strict sense, the environment, and thus incentives faced by the students under the quota system are different from those under quality standard system. Under fixed quota system, how other students will behave has a direct impact on the probability of my being selected. For example, at the margin, if I get admitted, then another student must be dropped out to keep the fixed quota. Partly because I have here a continuum of students and for the sake of simplicity, I rule out such strategic considerations and use the same FOC as is used in the earlier section. In other words, I am assuming that every student plays "against the field." For a model which attempts to explicitly consider the strategic aspects, see Dixit(1981).

First, let's take a look at the effects of changing ϕ, the new parameter which represents the admission rate, on t^* and e^*. Differentiating equations 1) and 2) in (29) with respect to ϕ generates the following.

$$Xf'\alpha\left(\alpha\frac{de^*}{d\phi}-\frac{dt^*}{d\phi}\right)-\beta c''\frac{de^*}{d\phi} = 0 \tag{29}$$

$$\int_{\underline{x}}^{\bar{x}} f\left(\alpha\frac{de^*}{d\phi}-\frac{dt^*}{d\phi}\right) dH(X) = 1. \tag{30}$$

Solving (29) for $de^*/d\phi$ generates,

$$\frac{de^*}{d\phi} = \frac{Xf'\alpha}{\mu}\frac{dt^*}{d\phi}. \tag{31}$$

Note that the coefficient of $dt^*/d\phi$ above is just the same as $\partial e^*/\partial t^*$ given in (11). Therefore, the following result holds,

$$\frac{de^*}{d\phi} = \frac{\partial e^*}{\partial t^*}\frac{dt^*}{d\phi}, \tag{32}$$

which is self-explanatory. That is, the question of how students will respond to an increase in admission rate hinges on how much of the cut-off point will be adjusted in response to the admission rate change, and how students will react to that cut-off point change.

Of course, the required change in t^* to make the new ϕ hold, in turn, depends on how students will behave in response to the ϕ change. So, the logics are circular, and we have to solve them simultaneously.

Inserting (31) into (30), and solving for $dt^*/d\phi$ yields,

$$\frac{dt^*}{d\phi} = \frac{1}{\int \frac{f\beta c''}{\mu} dH(X)} < 0. \tag{33}$$

Thus, the cut-off score must be cut down. This is, in a sense, a trivial result because in order to increase the number of students admitted, you have to lower the cut-off score with other things fixed.

By definition, an integral of individual student's winning odds over the whole population gives the fraction of students who are admitted, the admission rate, ϕ. Therefore, the whole expression in the denominator of equation (33) is simply $d\phi/dt^*$, the resulting change in admission rate when t^* is one unit increased. This can be again easily checked by directly differentiating equation 2) in (28) with respect to t^*. Since the sign of this expression is negative, it gives simply how many students will additionally fail if you increase the cut-off score by one point.

Plugging (33) back into (31) gives the effects of change in ϕ on individual student's effort level. Since the coefficient of $dt^*/d\phi$ is the same as the one given in (11), and $dt^*/d\phi$ is negative, it implies that those students whose expected scores are in excess of the cut-off score will work less, and students with low scores will work more as the admission rate increases. As it gets easier to gain an entry into the college due to the admission rate increase, those students with relatively high scores now work less hard, because it is no longer necessary to put that much effort to get in. Whereas, those students with lower scores work harder because their chance of being admitted has now been improved.

The fact that students will show different responses to the change in admission rate depending on his/her relative position to the cut-off score gives rise to the similar implications on the aggregate tutoring, discussed earlier. That is, the aggregate volume of private tutoring will crucially depend on the current state of competitiveness of college admission. If it is already highly competitive, relaxing it by increasing the admission quota a little bit will not help reduce the private tutoring, but aggravate the situation further.

Increasing the Cost of Private Tutoring

The effect of increasing the cost of private tutoring can be analyzed in a similar way. Differentiating equations 1) and 2) in (28) with respect to β gives,

$$Xf'\alpha\left(\alpha\frac{de^*}{d\beta}-\frac{dt^*}{d\beta}\right)-\beta c''\frac{de^*}{d\beta}-c'=0 \tag{34}$$

$$\int_{\underline{x}}^{\bar{x}} f\left(\alpha\frac{de^*}{d\beta}-\frac{dt^*}{d\beta}\right)dH(X)=0. \tag{35}$$

Solving (34) for $de^*/d\beta$ yields,

$$\frac{de^*}{d\beta}=\frac{Xf'\alpha}{\mu}\frac{dt^*}{d\beta}+\frac{c'}{\mu}. \tag{36}$$

Note that the second term in (36) is exactly the same as that in (23) above. So this is the direct effect of raising the cost of private tutoring on students' effort level, the sign of which is negative. The first term then takes into account the effect of cost increase on t^* and each student's subsequent adjustment in effort level due to the change in t^*. Simply put, the equation (36) can be rephrased as,

$$\frac{de^*}{d\beta}=\frac{\partial e^*}{\partial t^*}\frac{dt^*}{d\beta}+\frac{\partial e^*}{\partial\beta}. \tag{37}$$

Inserting (36) into (35) and solving for $dt^*/d\beta$ renders,

$$\frac{dt^*}{d\beta}=-\frac{\int\frac{\alpha fc'}{\mu}dH(X)}{\int\frac{f\beta c''}{\mu}dH(X)}<0. \tag{38}$$

The denominator is what we have already seen in (33) and is negative. In overall, an increase in tutoring cost will put a downward pressure on the cut-off score. The intuitive reason behind this result is straightforward. An increase in tutoring cost induces every student to take less tutoring, and this lowers the overall distribution of test scores. Under the situation, the school authority has to lower the cut-off point of the test in order to continue to secure a given fraction of students.

It is interesting to note that the numerator in (38) is just $d\phi/d\beta$

evaluated under the assumption of fixed t^*. That is, differentiating the equation 2) in (28) with respect to β, holding t^* constant, yields,

$$\frac{d\phi}{d\beta}\Big|_{dt^*=0} = \int \frac{\alpha f c'}{\mu} dH(X) < 0. \qquad (39)$$

So it captures the effect of changing β on the admission rate under the fixed t^*, and the sign is negative. As mentioned earlier, an increase in tutoring cost causes every student to take less tutoring. This lowers the expected score of every student. Consequently, if the school keeps the cut-off point at the old level ($dt^*=0$), the fraction of students who are admitted will be curtailed.

Cancelling this negative effect on the admission rate of increased cost and restoring the original admission rate requires a corresponding adjustment in the cut-off score in an opposite direction. The size of the latter adjustment necessary is obtained by dividing this expression by the denominator in (38) which happens to be equal to $d\phi/dt^*$. In short, the following relationship holds.

$$\frac{dt^*}{d\beta} = -\frac{\frac{d\phi}{d\beta}\Big|_{t^*=exog}}{\frac{d\phi}{dt^*}\Big|_{t^*=exog}} \qquad (40)$$

Note that this is the very relationship between t^* and β (or any other parameters for that matter) that should hold when you want to keep ϕ constant in equation 2) in (28).

Generalizing the above result, we obtain the following "fundamental relationship" between the cut-off score and every exogenous parameter of the model for maintaining a fixed admission rate. Denoting the representative exogenous parameter as z, we have,

$$\frac{dt^*}{dz} = -\frac{\frac{d\phi}{dz}}{\frac{d\phi}{dt^*}} = -\frac{\int \frac{dF}{dz}\mid_{t^*=exog} dH(X)}{\int \frac{dF}{dt^*}\mid_{t^*=exog} dH(X)}$$

$$= -\frac{\int [\frac{\partial F}{\partial e^*}\frac{\partial e^*}{\partial z} + \frac{\partial F}{\partial z}]\, dH(X)}{\int [\frac{\partial F}{\partial e^*}\frac{\partial e^*}{\partial t^*} + \frac{\partial F}{\partial t^*}]\, dH(X)}, \quad (41)$$

with an understanding that both numerator and denominator are to be evaluated at equilibrium point treating t^* as if exogenous[18]. Note that the denominator contains no expressions related to exogenous parameters. Consequently the denominator is not associated with any particular parameter, but is common to every exogenous parametric change.

The effect of cost change on each student's equilibrium effort level is now obtained by plugging (38) into (36). The overall effects on individual efforts are likely to be negative for those students whose expected scores are greater than the cut-off score since both the last term and $dt^*/d\beta$ in the first term in equation (36) are negative, and the coefficient of $dt^*/d\beta$ is positive for them. For those students whose expected scores are below the cut-off point, however, the sign is indeterminate because the whole first term is positive but the second term is negative.

In the final equilibrium after the adjustments both in students' effort levels and the school's cut-off score have been made, the change in individual student's final winning probability depends crucially on his/her position relative to the whole distribution. Some students' odds will improve, and others' not, depending on the criterion to be discussed shortly. This is one of the most important differences between the quality standard system and the quota system. In the earlier section where the cut-off point is exogenously set, the directions of changes in individual student's odds have been in most cases uniform across the board, as are given in (17), (18) and (25)[19].

Under the current quota system, however, if some students get extra

18) This explains why the expression, $\partial e^*/\partial z$, in the numerator has the partial derivative sign, and the expression, $\partial e^*/\partial t^*$, can be defined and have any meaning.

19) One exception has been the case in which the dispersion of the test scores has been increased. There some students get hurt while others benefit in terms of winning odds.

benefits due to a policy change, in terms of their winning odds, then some others must get hurt. This is simply because of the definition of the quota system. If someone gains, others must lose to keep the level flat[20]. The new cut-off score will be set exactly at the point that will accomplish this leveling task, somewhere in the middle that will balance off the opposing forces. And the resulting changes in individual student's odds should occur in such a way that the aggregate admission rate will not be affected. This is, of course, trivial in the sense we have already imposed such a condition from the beginning, but worthwhile to check whether it really holds and how.

Since the above claim applies to every parametric change, I will use the generalized version given in (41) in deriving the proof. Let the inside of the integral sign in the denominator be denoted as ξ. And then, by dividing and multiplying the inside of the integral of the numerator by ξ, we have the following relationship,

$$\frac{dt^*}{dz} = \frac{\int - \frac{\frac{dF}{dz}\Big|_{t^*=exog}}{\xi}\ \xi\ dH(X)}{\int \xi\ dH(X)}. \qquad (42)$$

Let the first term inside of the integral of the numerator be denoted by still another term K, so that we have,

$$\frac{dt^*}{dz} = \frac{\int K\,\xi\, dH(X)}{\int \xi\, dH(X)} = \overline{K}. \qquad (43)$$

Under this new representation, dt^*/dz , the change in equilibrium cut-off point due to a parametric change, can be interpreted as a weighted average of K, with the weights being given by ξ. Correspondingly, the generalized version of the equation (36), which represents the change in equilibrium effort due to a parametric variation, becomes,

20) Although I have introduced the assumption that every student is playing against the field, some remaining negative externalities are unavoidable, because they are inherent in the quota system.

$$\frac{de^*}{dz} = \frac{\partial e^*}{\partial t^*}\frac{dt^*}{dz} + \frac{\partial e^*}{\partial z} \tag{44}$$

$$= \frac{\partial e^*}{\partial t^*}\overline{K} + \frac{\partial e^*}{\partial z}.$$

Now, differentiating the winning odds function, F, with respect to z, and substituting (44) for dt^*/dz yields[21],

$$\frac{dF}{dz} = \frac{\partial F}{\partial e^*}\frac{de^*}{dz} + \frac{\partial F}{\partial t^*}\frac{dt^*}{dz} + \frac{\partial F}{\partial z} \tag{45}$$

$$= \frac{\partial F}{\partial e^*}\left[\frac{\partial e^*}{\partial t^*}\overline{K} + \frac{\partial e^*}{\partial z}\right] + \frac{\partial F}{\partial t^*}\overline{K} + \frac{\partial F}{\partial z}.$$

Collecting terms and further simplifying generates,

$$\frac{dF}{dz} = \left(\frac{\partial F}{\partial e^*}\frac{\partial e^*}{\partial t^*} + \frac{\partial F}{\partial t^*}\right)\left[\overline{K} + \frac{\frac{\partial F}{\partial e^*}\frac{\partial e^*}{\partial z} + \frac{\partial F}{\partial z}}{\frac{\partial F}{\partial e^*}\frac{\partial e^*}{\partial t^*} + \frac{\partial F}{\partial t^*}}\right] \tag{46}$$

$$= \xi\,[\overline{K} - K].$$

Equation (46) says that change in individual student's winning odds in the final equilibrium in the wake of a parametric change solely depends on his/her position relative to the population mean of K. Since ξ is negative, if his/her K is greater than the whole population mean, his chance of winning will improve, whereas if his/her K is smaller than the mean, his chance will deteriorate.

As mentioned earlier, it is redundant to check whether these improvements and deteriorations of individual student's winning odds will indeed average out among students, and, thus, consequently result in an unchanged admission rate, because we have solved everything so far under that condition. But it is still illuminating to confirm the result because it reassures us that all the necessary adjustments should be made in such a way that in the final equilibrium the

21) Note that in this paper we have the following three different but related expressions; dF/dz, $dF/dz\,|_{t^*=exog}$, and $\partial F/\partial z$.

admission rate should not be affected at all.

Differentiating ϕ with respect to z gives,

$$\begin{aligned}\frac{d\phi}{dz} &= \int \frac{dF}{dz}\, dH(X) \\ &= \int \xi[\,\overline{K} - K\,]\, dH(X) \\ &= \overline{K}\int \xi\, dH(X) - \int \xi\, K\, dH(X) \\ &= 0,\end{aligned} \tag{47}$$

where the last equality follows from the very definition of $\overline{K}$, given in (43). This proves the claim asserted in the above.

Now, we have to explicate what the K above really stands for. Looking back the expression in the second equality of equation (41) is helpful in this regard. The inside of the denominator given there is simply ξ. Therefore, K is just a ratio of the following two derivatives,

$$K = -\frac{\left.\dfrac{dF}{dz}\right|_{t^*=exog}}{\left.\dfrac{dF}{dt^*}\right|_{t^*=exog}}. \tag{48}$$

Given this, K is simply the Marginal Rate of Substitution (MRS) between z and t*, holding F constant. Thus, it gives the rate of change in t^* that is necessary to keep the winning odds of each student constant when there is a change in a parameter. If a student works harder in response to a particular parameter change, his/her odds will probably increase. To keep his/her odds at the old level, the school authority has to adjust the cut-off point a bit upward. If another student works less, then the school authority has to lower the cut-off point correspondingly for him/her.

Note that the adjustments of cut-off point in this hypothetical experiment are all individualized, targeted for each specific student. The school, however, cannot have multiple cut-off points, custom-tailored for each student. It has to set just one cut-off point applicable to every student somewhere in the middle considering the whole distribution of the required adjustments. So it must take a weighted average of those necessary adjustments. This is exactly what drives the results we have had in (43) and (46) above. This also explains why we should take t^* exogenous while evaluating every term in (41) and (48).

The weights used to compute the average, is $\xi = dF/dt^*$, the sensitivity of F to the changes in t*. Thus, $\overline{K}$ is the weighted average of the individual student's MRS between an exogenous parameter, z, and the cut-off point, t^*, with the weights being the sensitivity of F with respect to the cut-off point, t^*.

Lowering the Marginal Returns to Private Tutoring

It is now straightforward to analyze the effect of a change in α. All we need is just to follow the path that has been cleared in the above. Differentiating equation 1) in (28) with respect to α, and simplifying gives,

$$\frac{de^*}{d\alpha} = \frac{\alpha X f'}{\mu}\frac{dt^*}{d\alpha} + \frac{-Xf - \alpha X f' e^*}{\mu}. \tag{49}$$

Again, the second term in the right hand side is the direct effect of an increase in α on the effort level which has appeared in equation (14). As explained there, most students, except those with very high expected scores, will increase their efforts in response to the raised marginal returns to private tutoring. The first term is the indirect effects induced by the change in cut-off score the direction of which will be determined shortly.

Utilizing the results obtained in the above, and taking partial total derivatives of ϕ in equation 2) in (28) with respect to t^* and α, respectively, treating t^* exogenous, and lastly taking the negative ratio between the two gives,

$$\frac{dt^*}{d\alpha} = \frac{\int \frac{f\beta(c' + c''e^*)}{\mu} dH(X)}{\int \frac{f\beta c''}{\mu} dH(X)} > 0. \tag{50}$$

Enhanced marginal returns to private tutoring thus causes the cut-off score to move upward. The intuition behind this results is again fairly simple. An improvement in marginal returns to tutoring raises the expected score, αe^*, of every student, given e^*. Furthermore, almost every student chooses to take more effort, e^*, in response to the α increase. The combined effect of this two upward pressures causes the cut-off score to rise.

Plugging this result back into (49) will give the final equilibrium solution for $de^*/d\alpha$.

Increasing the Noise of the Test

Differentiating the equation 1) in (28) with respect to σ yields,

$$\frac{de^*}{d\sigma} = \frac{\alpha X f'}{\mu}\frac{dt^*}{d\sigma} + \frac{\partial e^*}{\partial \sigma}. \tag{51}$$

With every term in the RHS of (51), we are familiar now. The second term is the direct effect of σ change on the effort level, which we have had in (19). The first term is the indirect effect that comes through its effect on t^* which we will examine below.

In the previous section, we examined the effects of the increased noise of the distribution on the winning odds of each student at some length. There we have seen that the students can be grouped into four different categories; one group has improved winning odds with enhanced efforts, the second group has indeterminate odds with higher efforts. The third group has deteriorated odds with lowered efforts and the last group indeterminate odds with decreased efforts. Taking an integral of these changes, which are represented as $dF/d\sigma$ in (22) but now has to be interpreted as $dF/d\sigma|_{t^*=exog}$, over the whole population gives the change in admission rate due to the change in the variance of the density. This will work as a numerator in (41).

$$\begin{aligned}\frac{d\phi}{d\sigma}\Big|_{t^*=exog} &= \int \frac{dF}{d\sigma}\Big|_{t^*=exog} dH(X) \\ &= \int \left[F_\sigma - \frac{\alpha^2 f_\sigma X f}{\mu}\right] dH(X).\end{aligned} \tag{52}$$

Taking the negative of this, and dividing it by $d\phi/dt^*$, which is just the integral of (12) or, equivalently, the denominator of (41) itself, renders our desired solution for $dt^*/d\sigma$, the size of the required adjustment in t* that would keep the admission rate constant in the wake of the increased dispersion,

$$\frac{dt^*}{d\sigma} = \frac{\int [-F_\sigma + \frac{\alpha^2 f_\sigma X f}{\mu}] \, dH(X)}{\int \frac{f\beta c''}{\mu} \, dH(X)} . \tag{53}$$

For the reasons explained already in the previous section, its sign is indeterminate. Depending on the location of the cut-off point, or, alternatively, the degree of competition in college admission, the sign of the whole expression may be positive or negative.

4. Conclusion

Rent seeking is pervasive in many controlled markets. In the education market, where tuition control is routinely practiced, this rent seeking is more serious than in other market. The divergence between the value a student is willing to pay and the price he has to pay to get one unit of education creates an exploitable rent, and thus incentives for every potential students to fiercely compete for admission ticket.

The form and volume of rent seeking will crucially depends on the selection criterion adopted, its effectiveness and precision in the measurement and policy measures to cope with them. Two general forms of selection criteria are considered in the paper; a quality standard system under which those with higher test scores than a preset cut-off value are admitted and a quota system under which those with higher rank are admitted.

Under the quality standard system, the overall effectiveness of policy measures aimed to reduce rent seeking, say, reducing the marginal efficacy of private tutoring, increasing the cost of tutoring, or increasing noise elements in assessment will critically hinge on the current degree of competitiveness of the market. If it is highly competitive, a reduction of the cut-off score will not reduce the rent seeking, but will encourage the rent seeking further. Any measures to increase the dispersion of test score, thus making it less reliable means to improve his/her test scores will not contribute to the reduction of rent seeking activity, either. Rather, making the market more competitive by raising the cut-off standard or by reducing the quota may sometimes be generate better results as far as a reduction of private tutoring is concerned.

Under the quota system, almost the same conclusions would hold, with regard to competition rate in the market. However, in addition to this, quota system inevitably creates negative externalities among students in the sense that an improvement in one contender's winning odds must be purchase at the cost of another contender's loss of his/her odds. And because of this, some one's gain should be matched by some other's loss. Depending on who becomes the new winner and who becomes a induced loser, in the wake of policy shift, allocative efficiency can be improved or deteriorated.

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