

The Use of the Term Structure of Interest Rates as a Target of Monetary Policy in an Economy with Frictions

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Abstract

Our paper explores a transmission mechanism of monetary policy through bond market. Based on the assumption of delayed responses of economic agents to monetary shocks, we derive a system of equations relating the term structure of interest rates with the past history of money growth and test the equations with the US data. Our results confirm that monetary policy targeting a specific shape of the term structure of interest rates could be implemented only with a certain time lags due to path-dependency of interest rates.

1 Introduction

The primary purpose of our paper is to investigate the roles of monetary policy in shaping the term structure of interest rates. The roles of money are defined in various ways. Among the most critical ones are the roles of money as an accounting unit, a store of value, and a medium of exchange. Due to these particular functions of money, monetary policy governing the stock of money, influences the relative prices of money delivered at different time and different states. In turn, the current relative prices of money defined in the space of time and states, which is, in other words, called the term structure of interest rates, influence economic decisions by private agents. Thus, a thorough exposition of monetary policy would encompass the analysis of a monetary general equilibrium model including all of the above mentioned processes. However, for brevity, we narrow down the scope of this paper to showing how the monetary policy can manipulate diverse interest rates along the passage of time.

Intuitively speaking, the term structure of interest rates is much more informative than any single economic variables and thus will be useful as a criterion for policy making. So far there have been continuous debates over what should be optimal targets of monetary policies. Mostly a combination of inflation and GDP gap is cited as a candidate for the target of monetary policy (Taylor(1993)). Further developed models allow autoregressive formations in inflation and GDP gap (Clarida et al.(2000)). Based on such criteria, a certain level of short-term interest rate (e.g. call rate in Korea, federal fund rate in the US) is prescribed that a central bank should maintain.

Apart from the tradition, our paper argues that an effective monetary policy should consider the whole term structure of interest rates rather than a yield rate of a bond with specific maturity. Furthermore, though the control over the short-term interest rate has influence on the yields of bonds with longer maturities, it has not yet been clearly verified in which direction a change in the short-term interest rate shifts the whole term structure of the interest rates, which requires thorough understanding of the whole economy.

Frequently we read numerous articles about predicting the future path of federal fund rate from newspapers. All of them are written on the implicit belief that monetary policies have influence on major aggregate economic activities, such as consumption and investment, though their influence on the economic activities may differ in terms of directions, magnitudes, and timing. Unfortunately, a true transmission mechanism of monetary policies has not yet been thoroughly explored. However, the transmission mechanism works through multi-channels, only one or two of which have so far been highlighted. In our knowledge, very few economic models emphasize the effects of monetary policies in the context of analyzing an equilibrium of the whole nominal bond market.

On the other hand, from the literature on consumption and investment, we already know that consumption of durable goods and investment is quite sensitive to economic fluctuations in comparison with consumption on non-durable goods and services. Intuitively speaking, since the flows of benefit from durable goods and capital continue for a certain period of time, durable good consumption and investment entail the feature of irreversibility or indivisibility of purchase, which reduces durable goods consumption and investment decisions to optimal stopping problems. For the reason, it is absurd to expect that the monetary authority can control the economy by merely changing the short-term interest rate. In reality, the falling short-term interest rate is often accompanied by an increase in the long-term interest rate, which may discourage an agent from purchasing durable goods and physical capital. Thus, the monetary authority should find a certain

pattern of a yield curve in order to reset the current yield curve to the pattern and boost the economy in times of depression .

Our paper proposes to (1) investigate how a monetary policy (not only quantity-easing but also targeted at controlling the short-term interest rate) shifts the whole term structure of interest rates, (2) discuss the implications of the observation that consumption on durable goods and investment are sensitive to changes in the term structure of interest rates, and (3) arrange monetary policies of maintaining a certain shape of the term structure of interest rates based on empirical results.

The contents of the paper are organized as follows: Section 2 discusses a transmission channel of monetary policy in the economy, which relies on the lagged adjustment processes of various interest rates in the bond market. The feature of lagged adjustments resulting from delayed responses to monetary shocks is critical in that it relates the dynamics of interest rates to the past history of money growth rates. Section 3 tests all the hypotheses obtained from the models introduced in section 2 using the US data, both monthly and quarterly. The relationship between the term structure of interest rates and the money growth rates are estimated in presence of as well as in absence of exogenous production fluctuations. Section 4 deduces the policy implications mainly emphasizing that the monetary authority can improve its performance by substituting the term-structure targeting policy for the current short-term interest rate targeting policy. Finally section 5 concludes.

2 Theoretical Consideration

From a survey of the existing literature on the optimal monetary policy, we identify two common approaches from two distinctive traditions of thought-New classical and New Keynesian. New Classical approach admits that market incompleteness, such as market segmentation, may cause the effect of monetary policy whereas New Keynesian approach introduces sticky prices and wages to refute the neutrality of money. In essence both approaches have common in exploiting implicit or explicit delayed responses of economic agents. This section is purposed to provide a logical explanation about the delayed responses of aggregate macro variables to monetary shocks and reveal the consequences of the delayed responses on the dynamics of monetary policy.

To begin with, we investigate a limited bond market participation model and show that the higher order moments of money supply can influence the term structure of interest rates. Extended

from a traditional *Cash-in-Advance* model of Lucas and Stokey(1987), a general *m-period-ahead* *CIA* condition is imposed. The adoption of *CIA* feature is critical because it, combined with the assumption of limited bond market participation, brings about redistributive and persistent effect of monetary policies on the economy. From the assumptions, it is deduced that the term structure of interest rates is approximated by a system of linear equations of the lagged money growth rates. As is generally understood (Clarida et al.(2000) and Lucas(2000)), the expectation of the future money growth rates (or the future monetary policy) have effect on the current term structure of interest rates. However, we emphasize the importance of the past paths of monetary expansion in a sense that money shock would be realized in a differential manner across agents of the economy.

Second, we explore the implications the non-negativity restriction of nominal bond yield rates holds in financial market, while showing that our approximation of the term structure of interest rates by the past money growth paths does not necessarily satisfy the non-negative condition. The non negativity restriction of nominal bond rate is a critical ground for the central bank to consider when it exercises open market operation policy. Especially, in a low inflation state, the possibility of reaching zero short-term interest rate is often worried because zero rate is regarded as a natural lower boundary of so called liquidity trap and it is commonly believed that the monetary policy without coordination with the expansionary fiscal policy would be ineffective in such a situation. Hence, we discuss the effectiveness of monetary policy at a near zero short-term interest rate and explore the ways of enhancing the performance of monetary policy.

Third, we examine a claim that consumption and investment decisions are significantly affected by the term structure of interest rates while the demands for durable goods and capital goods are more sensitive to a change in the term structure of interest rates than consumption of non-durable goods and services due to their (longer) duration of usage. All other things equal, lower short-term interest rate is likely to induce more current consumption. However, what if the lowered short-term interest rate is matched by higher long-term interest rate? A casual answer we can get without considering the dynamics of the term structure would be that the effect of lowering the short-term interest rate on the consumption is ambiguous. Thus, we also aim to answer for a question why a monetary policy targeting a certain level of the short-term interest rate leads to different economic performances at different times.

Nominal bonds, which guarantee the delivery of pre-defined amount of money at maturities,

are (gross) substitutes for money ¹. Private agents allocate their resources between money and nominal bonds ². Thus, a change in money stock implies that the economy should move to another equilibrium sustaining different relative prices of bonds with respect to money. This section focuses on analyzing a mechanism, through which variations in monetary policy lead to different term structure of interest rates. A basic idea that not only the expectation of the future money growth rates but also the past money growth path determine the current term structure of interest rates, would explain why it leads to different outcomes to maintain the same level of the short term interest rate at different periods ³.

Needless to say it would be another paper topic to verify how the term structure of interest rate can have real effects on the economy. The emphasis on the relationship of the term structure of interest rates and real macro variables is originated from our original intention to transform the issue of finding optimal monetary policy to that of finding an appropriate term structure. This issue has been tried commonly from the stance of so called New Keynesianism (though at a slightly different angle from ours). However, in this paper, we do not delve into this issue further. Instead we concentrate on revealing the relationship between money and the term structure of interest rates.

2.1 Lagged transmission channel

In this section we introduce an economy with limited bond market participation in order to induce a situation in which a monetary shock has differential impacts on heterogeneous agents across time (mainly redistributive effects). The impact differential is caused by the different timing of money shock transmitted to the agents or their different speed of reactions to the shock and it leads to a non-trivial pattern change in the term structure of interest rates. Otherwise, the yield curve goes flat or it depends only on the expectation of the future monetary growth path and real macro variables. Of course, it is reasonable that the yield curve is influenced by the future policy variables. However, we have no clue as to how the accumulation of the information on the past history is

¹In other words, money is a nominal bonds, which expires instantly.

²In fact, nominal bonds vary not only by the length of maturities but also by the magnitude of default risk. However, in the paper, for simplicity we only deal with government issued bonds. The status of the government as a sole provider of currency in the economy eliminates default risk premium of the government bonds.

³The expectation for the future can be understood as reflection of the past. In this sense the persistent effect of the past policy can be more substantial.

reflected on the formation of the expectation for the future. Thus, we need a model where the term structure of interest rates depends on the past history of monetary policy as well as the future policies ⁴.

Our model is an adapted version of Alvarez, Lucas and Weber (2001). Consider an economy where exist two types of agents-bond-market participant and non-participant. Regardless of the type, both group have the same intertemporal utility function.

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t U(C_t), \text{ where } U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \quad (1)$$

λ portion of the population participates in bond trading and the $(1-\lambda)$ portion does not. The aggregate production of this economy is y_t .

$$y_t = \lambda C_t^T + (1-\lambda) C_t^N + \frac{T_t}{P_t}, \quad (2)$$

where C_t^T and C_t^N are consumption of the trader and the non-trader each and T_t is the nominal value for lump-sum tax payment. The budget constraint for the non-trader is

$$P_t C_t^N = \sum_{j=0}^m \nu_{t-j,t} P_{t-j} y_t, \text{ where } \sum_{j=0}^m \nu_{t,t+j} = 1. \quad (3)$$

⁴One weakness of our model is that it doesn't consider the effect of monetary policy on production. The description of the production sector and its interactions with monetary policy are omitted because the introduction of a production function in the economy involves the calculation of a steady state and discussions on transitional paths. True that the interactions of the yield curve with production as a possible channel of monetary policy, is crucial, we do not pursue towards the direction in the paper. For a more general setup, an aggregate supply function in consideration of the delayed effects could be inserted as follows:

$$\pi_t = \sum_{i=0}^u \kappa_i E_{t-i} x_t + \sum_{i=1}^z \zeta_i \pi_{t-i} + \sum_{i=1}^k \gamma_i E_{t-d} \pi_{t+1},$$

where x_t is a GDP gap at time t .

Another point to mention is regarding the policy time lag. Following simple Keynesian logic about the effect of monetary expansion, the expansion lowers the short-term interest rate and the lowered interest rate encourages consumption and investment. The increased aggregate demand spurs the production and lowers the unemployment rate, leading to the GDP growth. It is at this point that the policy time lag argument intervenes. The above logic implicitly assumes that it takes some time in changing hands of money holders for the transmission channel to be in effect. In contrast our model conjectures that the policy time lag arises because it takes a certain length of time for a new policy to change a signal or information, on which private agents make economic decisions. This new interpretation of policy time lag, we believe, is more valid when markets (especially financial markets) respond fast to external shocks.

At each period he sells his product in the market and receives cash in return ($P_t y$). He allocates this proceeds across $m + 1$ periods on consumption with the proportion of $\nu_{t,t+j}$, $j = 0, 1, \dots, m$. Another more realistic interpretation of this *m-period-ahead CIA* feature is that $\nu_{t,t+j}$, $j=0,1,\dots,m$, is the proportion of consumers who need j period time lag in responding to monetary shocks.

On the other hand, the trader spends his money not only on consumption but also on bond trading.

$$\begin{aligned} P_t C_t^T &= \sum_{j=0}^m \nu_{t-j,t} P_{t-j} y_t + \frac{1}{\lambda} \left(B_t - \left(\frac{1}{1+r_t} \right) B_{t+1} - T_t \right) \\ &= \sum_{j=0}^m \nu_{t-j,t} P_{t-j} y_t + \frac{M_t - M_{t-1}}{\lambda} \end{aligned} \quad (4)$$

Bond and money supplies satisfy

$$B_t - \left(\frac{1}{1+r_t} \right) B_{t+1} - T_t = M_t - M_{t-1}, \quad (5)$$

where the government levies the lump-sum tax T_t on the trader only. The goods market equilibrium is attained when the next equation holds:

$$P_t C_t = P_t C_t^N + P_t C_t^T = P_t y_t \quad (6)$$

Combining (2)-(6), we obtain

$$\begin{aligned} P_t y_t - T_t &= \sum_{j=0}^m \nu_{t-j,t} (P_{t-j} y_{t-j}) + M_t - M_{t-1} \\ &= M_{t-1} + \nu_{t,t} P_t y_t + M_t - M_{t-1} \\ &= \nu_{t,t} P_t y_t + M_t. \end{aligned} \quad (7)$$

Accordingly, the equation of exchange is written as

$$M_t \frac{1}{1 - \nu_{t,t}} = P_t y_t. \quad (8)$$

Thus, $\nu_{t,t}$ can be understood as the money velocity.

From the above equations, we represent the consumption of the trader in the function of money growth rates. Here it is noteworthy that we are interested in the consumption of the trader because in the bond market only the marginal utility of the trader matters for the determination of a yield curve.

$$C_t^T = \frac{1}{\lambda} y_t - \frac{(1-\lambda)}{\lambda} C_t^N - \frac{T_t}{P_t} = \frac{1}{\lambda} y_t - \frac{(1-\lambda)}{\lambda P_t} \sum_{j=0}^m \nu_{t-j,t} P_{t-j} y_{t-j} - \frac{T_t}{P_t} \quad (9)$$

where $g_t = \frac{T_t}{P_t}$, $\mu^t = (\mu_t, \dots, \mu_{t-m})$, and $v^t = (\nu_{t,t}, \dots, \nu_{t-m,t})$. Then, the equilibrium nominal interest rate must satisfy the following marginal condition

$$\left[\frac{\frac{1}{1+r_{t,t+k}}}{\frac{1}{1+\rho}} \right]^k = E_t \left[\frac{U' (c(\mu^{t+k}, v^{t+k}) y - g_{t+k})}{U' (c(\mu^t, v^t) y - g_t)} \frac{\frac{(1-\nu_{t,t})}{(1-\nu_{t+k,t+k})}}{\prod_{i=0}^k (1 + \mu_{t+i})} \right] \quad (10)$$

Notably, $c(\mu^t, v^t)$ appears to be independent of g_t . However, it does not imply that g_t has no effect on the determination of $c(\mu^t, v^t)$. Rather, the effect of g_t on $c(\mu^t, v^t)$ is hidden in the terms (μ^t, v^t) by medium of (5).

For simplicity, we assume $y_t = y$ and $g_t = 0$ for all t . Then,

$$\begin{aligned} C_t^T &= \frac{1}{\lambda} y \left\{ 1 - (1 - \lambda) \sum_{j=0}^m \nu_{t-j,t} \frac{P_{t-j}}{P_t} \right\} - \frac{T_t}{P_t} \\ &= \frac{1}{\lambda} y \left\{ 1 - (1 - \lambda) \sum_{j=1}^{m-1} \nu_{t-j+1,t} \frac{1}{\prod_{i=1}^j (1 + \mu_{t-i+1})} \frac{(1 - \nu_{t,t})}{(1 - \nu_{t-j,t-j})} \right\} - g_t \\ &\equiv c(\mu_t, \dots, \mu_{t-m}, \nu_{t,t}, \dots, \nu_{t-m,t}) y - g_t \equiv c(\mu^t, v^t) y - g_t, \end{aligned} \quad (11)$$

$$\begin{aligned} \left[\frac{\frac{1}{1+r_{t,t+k}}}{\frac{1}{1+\rho}} \right]^k &= E_t \left[\frac{U' (c(\mu^{t+k}, v^{t+k}) y)}{U' (c(\mu^t, v^t) y)} \frac{1}{\prod_{i=0}^k (1 + \mu_{t+i})} \frac{(1 - \nu_{t,t})}{(1 - \nu_{t+k,t+k})} \right] \\ &= E_t \left[\left(\frac{c(\mu^{t+k}, v^{t+k}) y}{c(\mu^t, v^t) y} \right)^{-\gamma} \frac{1}{\prod_{i=0}^k (1 + \mu_{t+i})} \frac{(1 - \nu_{t,t})}{(1 - \nu_{t+k,t+k})} \right] \end{aligned} \quad (12)$$

Taking the first order approximation of $\log c(\mu^t, v^t)$ around the point $(\underline{\mu}, \underline{\nu})$, we obtain

$$\log c(\mu^t, v^t) \cong \left(\frac{1-\lambda}{\lambda} \right) \left\{ \sum_{j=0}^m \bar{\nu} \left(\sum_{i=0}^j \mu_{t-i} \right) + f(\nu^t) \right\} \quad (13)$$

$$U' (c(\mu^t, v^t) y) = \exp \left[-\gamma \left(\frac{1-\lambda}{\lambda} \right) \left\{ \sum_{j=0}^m \bar{\nu} \left(\sum_{i=0}^j \mu_{t-i} \right) + f(\nu^t) \right\} \right] y^{-\gamma} \quad (14)$$

After substituting (14) into (12) and taking log by both sides, then we obtain

$$\begin{bmatrix} r_{t,t+1} \\ r_{t,t+2} \\ \dots \\ r_{t,t+n-1} \\ r_{t,t+n} \end{bmatrix} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \phi_{1,3} & \dots & \phi_{1,m-1} & \phi_{1,m} \\ \phi_{2,1} & \phi_{2,2} & \dots & \dots & \dots & \phi_{2,m} \\ \phi_{3,1} & \dots & \dots & \phi_{i,j} & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \phi_{n-1,m} \\ \phi_{n,1} & \phi_{n,2} & \dots & \dots & \phi_{n,m-1} & \phi_{n,m} \end{bmatrix} \begin{bmatrix} \mu_t \\ \mu_{t-1} \\ \dots \\ \mu_{t-m+1} \\ \mu_{t-m} \end{bmatrix} + R(v^t) \quad (15)$$

or simply

$$\Gamma_t = \Phi_t \Delta_t + R(v^t), \quad (16)$$

where R_t and Δ_t are $n \times 1$ vectors, and Φ_t a $n \times m$ matrix.

The coefficients of the matrix in (15) are derived from (13) or (14). For $1 \leq j < m - i + 1$,

$$\phi_{i,j} = -\gamma \left(\frac{1-\lambda}{\lambda} \right) \bar{\nu} \quad (17)$$

For $m \geq j \geq m - i + 1 \geq 1$,

$$\phi_{i,j} = \gamma \left(\frac{1-\lambda}{\lambda} \right) \bar{\nu} \left\{ \frac{m-j+1}{i} \right\} \quad (18)$$

Neglecting that the expectation for the future monetary policies does not change, then the coefficients of Φ_t indicates that an increase in μ_t lowers the yield rates of bonds with shorter maturities than m periods while the yield rates of the bonds with maturities longer than m are raised. Combining these two, we can deduce that there is a slope change in the yield curve between m and $m+1$. Accordingly, the liquidity effect view is supported for bonds with maturities of $\{0, 1, \dots, m\}$ and the Fisher's view is valid for bonds with maturities of $\{m+1, \dots\}$.

A sudden change in μ_t has influence on the term structure of interest rates at time t in the following way:

$$\begin{bmatrix} dr_{t,t+1} \\ dr_{t,t+2} \\ \dots \\ dr_{t,t+n-1} \\ dr_{t,t+n} \end{bmatrix} = \Phi_t \begin{bmatrix} d\mu_t \\ 0 \\ \dots \\ 0 \\ 0 \end{bmatrix} + R(v^t) = \begin{bmatrix} \phi_{1,1} \\ \phi_{2,1} \\ \phi_{3,1} \\ \dots \\ \phi_{n,1} \end{bmatrix} d\mu_t + R(v^t) \quad (19)$$

Due to the delayed responses, shocks from monetary policy changes persist in the economy for a certain period of time. Accordingly, in order to discuss the effect of monetary policy on the term structure of interest rate, we have to consider the history of monetary expansion $\{\mu_{t-j}, j = 0, 1, 2, \dots, m\}$ ⁵. However, this does not guarantee that the central bank can achieve any arbitrarily shaped term structure, which is possible only when $m \geq \dim \Phi_t > n + 1$ (m : the policy time lag). Even if fine tuning is not possible, it would be useful to apply the least square method to achieve a certain shape of the term structure as below⁶.

$$\Delta_t^* = [\Phi_t' \Phi_t]^{-1} \Phi_t' [\Gamma_t^* - R(\nu^t)] \quad (20)$$

Numbers from (17) and (18) are obtained from the extremely restricted environment and are to be used for understanding the relationship between money growth and the changes in the term structure of interest rates. For empirical tests, modified versions of (16) could be applied without maintaining the restrictions of (15) and (16).

2.2 Liquidity trap and zero lower boundary

Equation (16) is obtained from the first order approximation and it may violate the non-negativity of nominal interest rates. Nominal interest rate is the return on holding nominal bonds. Due to the definition and the existence of money, we know that zero is a lower boundary for the nominal interest. However, it is not clear what would happen in the economy if the nominal interest rate approaches zero. Here we answer for questions whether zero or negative nominal interest is achievable and the zero nominal interest rate is a natural boundary for so called liquidity trap.

To begin with, we have to clarify why people hold money over other assets, which pay positive returns. There have been many trials to explain motives for holding money. Among them, the following three are commonly used.

(1) *Cash-In-Advance* model perceives money as a medium of exchange. Thus, all the economic transaction should be accompanied by money transaction in the opposite direction.

(2) *Risk free asset argument* highlights that there is no risk-free asset but money in the economy.

⁵ $\{\mu_{t-j}, j = 0, 1, 2, \dots, m\}$ in Equation (16) reflects path dependency of the term structure of interest rates. However, in another sense, it could be understood as the information on the higher moments of money growth rates.

⁶The non-singularity of $\Phi_t' \Phi_t$ is a critical condition to consider but we will not discuss the issue further here.

(3) *Money-in-the-utility* model assumes that money holding or real balance of money holding gives utility to a holder as physical commodities do.

We are not sure which of the above theories is more closer to the truth and there may be another way of explaining money holding. Anyway we can agree at least that money is held by economic agents. Furthermore, we regard money as a nominal bond with the shortest maturity, *i.e.* it matures every instant.

Consider the following markets: the money market with M_t amount of fiat money, government issued real bond market transacting $\{b_{t,t+n}, n=1, 2, \dots\}$, and government issued nominal bond market $\{B_{t,t+n}, n=1, 2, \dots\}$. $R_{t,t+n}$ and $r_{t,t+n}$ denote nominal and real yield rate of nominal and real bonds with n -maturity respectively.

Applying no-arbitrage condition, we could deduce the following results.

Proposition 1 $R_{t,t+n} = \max[r_{t,t+n} + \frac{\sum_{i=1}^n \pi_{t+i}}{n}, 0]$

Proof. $(1 + R_{t+k})^k = \prod_{i=1}^k (1 + R_{t+i-1,t+i}) = \prod_{i=1}^k (1 + r_{t+i-1,t+i} + \pi_{t+i}) = (1 + r_{t,t+k})^k \prod_{i=1}^k \pi_{t+i}$
if $R_{t+k} \geq 0$. ■

Proposition 2 If $r_{t,t+n} + \frac{\sum_{i=1}^n \pi_{t+i}}{n} \leq 0$, then the market for n -period nominal bond collapses *i.e.* $B_{t,t+n} = 0$.

Thus, we see that $R_{t,t+n}$ is always positive, and it is not defined when $r_{t,t+n} + \frac{\sum_{i=1}^n \pi_{t+i}}{n} \leq 0$ because $B_{t,t+n} = 0$.

From the above discussion we find that the yield rate should be positive if the issued amount of nominal bond is positive ($R_{t,t+n} = 0 \implies B_{t,t+n} = 0$). To rephrase, it implies that zero nominal interest rate makes the bond disappear from the market. Therefore, once the yield rate of a bond hits zero boundary, a monetary policy targeting the specific bond would become meaningless and the government or the monetary authority should redirect its attention to other kinds of bonds. Otherwise, so called a liquidity trap would hinder the economy from taking expansionary pressure from money side.

In a low inflation economy, the advent of liquidity trap is more worried than other cases. It is commonly believed that the government's open market operation is ineffective as the short-term interest rate moves near zero. However, this argument has logical flaw in that it simplifies the asset market to the substitution of money and one type of bond, whose rate of return in most cases is

the short-term rate. Accordingly, in this economy the near zero short-term interest rate signals that the bond market is just about to collapse and further injection of money into the economy cannot not boost the economy. However, in reality, there are various types of bonds, which differ by maturity and default risk. Thus, an increase in money stock will change interest rates of bonds with different maturities and have expansionary effect on the economy.

Open market operation by the monetary authority brings about different impacts on the economy depending on the maturities of bonds it deals with. Thus, the monetary authority should consider what maturity bonds it should target. By doing so, it chooses an intended type of the term structure of interest rates and the policy time lags to the targeted term structure (Orphanides(2003)⁷).

2.3 Decisions on consumption and investment

In this section we discuss the relationships of the term structure of interest rate to consumption and capital investment. The term structure of interest rates matters because consumption and investment decisions are the functions of the term structure of interest rates (Hong(1996 and 1997)⁸). Especially, durable good consumption and investment are defined to have positive signs when an optimal stopping time occurs. Of course the optimal stopping time is a function of the term structure, too.

⁷In the paper, Orphanides appreciates the usefulness of the open market operation policy, which is to “implement additional monetary expansion by shifting the targeted interest rates to that on successively longer-term instruments, when additional monetary policy easing is warranted at near zero interest rates”.

⁸Hong compares the sensitivities of price and interest rate on durable goods consumption using the US data and shows that durable goods consumption reacts sensitively to the price change but not to the variations in the interest rate. His interpretation is that the price reflecting the long-term forecast is more influential in determining durable goods consumption than the interest rate of the short-term forecast. The linkage of his idea to this paper is in that the price of durable good is the discounted sum of the future benefit flows by the term structure of interest rates.

$$\begin{aligned}
C_t^{ND} &= C_t^{ND} [w, r_{t,t+1}, r_{t,t+2}, \dots, r_{t+n}] \\
C_t^D &= C_t^D [D, w, r_{t,t+1}, r_{t,t+2}, \dots, r_{t+n}] \quad \text{if } t = \tau [r_{t,t+1}, r_{t,t+2}, \dots, r_{t+n}, D] \\
&= 0 \quad \text{if } t \neq \tau [r_{t,t+1}, r_{t,t+2}, \dots, r_{t+n}, D] \\
I_t &= I_t [K, w, r_{t,t+1}, r_{t,t+2}, \dots, r_{t+n}] \quad \text{if } t = \tau \\
&= 0 \quad \text{if } t \neq \tau [r_{t,t+1}, r_{t,t+2}, \dots, r_{t+n}, K]
\end{aligned}$$

Next, we discuss the volatilities of durable good consumption and capital investment. Unlike consumption of non-durable goods, investment and non-durable good consumption show more fluctuations in response to economic shocks including interest rate changes. By nature, decisions on durable good consumption and investment are very close to discrete choice or exercise of real options due to indivisibility of durable goods and physical capitals. Then, what indices does a decision maker refer to when he decides to purchase a durable good or physical capital?

First, consider a consumer's maximization program:

$$\begin{aligned}
&\max_{d_t} E \sum_{t=0}^{\infty} \beta^t U(C_t, D_t) \\
D_t &= (1 - \delta) D_{t-1} + d_t, \delta \in (0, 1) \\
A_{t+1} &= (1 + r_t) (A_t + Y_t - P_t C_t - Q_t d_t),
\end{aligned}$$

where P_t is the price of non-durable goods at t and Q_t is the price of durable goods at t . A_t and D_t are the amounts of financial assets and durable goods respectively the agent holds at time t . The value function of the agent is represented by the state variables A_t and D_t .

$$V[A_t, D_t] = U(C_t, D_t) + \beta E_t[V[A_{t+1}, D_{t+1}]]$$

The FOCs are

$$\begin{aligned}
\{C_t\} &: U_C(C_t, D_t) = P_t E[\beta(1 + r_t) V_A(A_{t+1}, D_{t+1})] \\
\{d_t\} &: U_D(C_t, D_t) = Q_t E[\beta(1 + r_t) V_A(A_{t+1}, D_{t+1})] - \beta E[V_D(A_{t+1}, D_{t+1})]
\end{aligned}$$

The envelope theorem leads to

$$\begin{aligned}
V_A(A_t, D_t) &= E[\beta(1 + r_t) V_A(A_{t+1}, D_{t+1})], \\
V_D(A_t, D_t) &= U_D(C_t, D_t) + \beta(1 - \delta) E[V_D(A_{t+1}, D_{t+1})]
\end{aligned}$$

Thus, the recursive manipulation of the above equations reveals

$$\begin{aligned}
V_A(A_t, D_t) &= \beta E \left[\prod_{i=0}^n (1 + r_{t+i}) V_A(A_{t+i+1}, D_{t+i+1}) \right], \\
V_D(A_t, D_t) &= U_D(C_t, D_t) + \beta (1 - \delta) E[V_D(A_{t+1}, D_{t+1})] \\
\frac{1}{\beta (1 + r_t) P_t} &= E \left[\frac{U_C(C_{t+1}, D_{t+1})}{U_C(C_t, D_t)} \right], \\
\frac{U_D(C_t, D_t)}{U_C(C_t, D_t)} &= \frac{Q_t}{P_t} - \frac{1}{P_t (1 + r_t)} \frac{E[V_D(A_{t+1}, D_{t+1})]}{E[V_A(A_{t+1}, D_{t+1})]}
\end{aligned}$$

Due to concavity of the utility function, an agent prefers to smooth cross-time allocation of consumption. Thus, he prefers to schedule (C_t, D_t) evenly. However, the gross change of D_t , d_t can adjust itself more flexibly. In other words, d_t moves with the greater volatility in order to guarantee the smoothing of D_t (the first difference of a variable is more volatile than the variable itself.). Hence, when we consider a firm's investment decision, we have to focus on the partial irreversibility of investment, which is partly related to durability.

Summing up, the private agents make decision based on both the future cash flows and the interest rate movement. The future path of interest rates is evaluated in the market in terms of yield rates of bonds with different maturities, which links consumption-investment to the term structure of interest rates. Channel, through which monetary policy affects the economy, may be numerous. However, the channel through the bond market is the most direct but least mentioned one.

3 Empirical Analysis

This section verifies the validity of the claims deduced in the previous section. The equation (16) implies that the dynamic system of the term structure of interest rates is governed by the past money growth rates, though the dynamic causality between the term structures of different periods is not explicitly displayed. In this section, mainly we use several modifications of (16) for empirical analysis.

3.1 Data

Our analysis is based on the US data from July 1959 to February 2000. We use the US data because the US government bond market is the most developed one and the maturities as well as

the volume of the bonds traded in the market are diverse and huge enough to plot a yield curve.

The variables of our concern are money stock, price and income variables in addition to interest rates. For the interest rates, we use federal fund rate, 3-month treasury bill, 6-month treasury bill, 1-year treasury bill, and a composite of long-term U.S. government securities. The composite of the long-term treasury bonds is specifically defined to be an unweighted average on all outstanding bonds neither due nor callable in less than 10 years. For the macro variables, we use M1 for an index of money stock, GDP deflator for price index, and real and potential GDP for an income measure.

The data frequencies differ from a category to another. For example, all the interest rates and M1 are recorded monthly whereas GDP deflator and GDP are recorded quarterly. To handle with the conflicts in the data frequencies at the same time exploiting the benefit of using monthly data, we run slightly different models both with monthly and quarterly data. In addition, we note that interest rates are measured in annum whereas M1, GDP deflator, and GDP measures are on a quarterly basis.

3.2 Results

To begin with, we define the following notations for brevity.

$$\Gamma_t = \begin{bmatrix} r_{t,t+1} \\ r_{t,t+2} \\ \dots \\ r_{t,t+n-1} \\ r_{t,t+n} \end{bmatrix} \in R_+^n, \Delta_t = \begin{bmatrix} \mu_t \\ \mu_{t-1} \\ \dots \\ \mu_{t-m+1} \\ \mu_{t-m} \end{bmatrix} \in R^{m+1}, \Lambda_t = \begin{bmatrix} y_t \\ y_{t-1} \\ \dots \\ y_{t-l+1} \\ y_{t-l} \end{bmatrix} \in R^{l+1}$$

$$\Gamma_t = A_1\Gamma_{t-1} + A_2\Gamma_{t-2} + A_3\Gamma_{t-3} + \dots + A_q\Gamma_{t-q} + R + \epsilon_t, \quad (21)$$

$$\Gamma_t = A_1\Gamma_{t-1} + A_2\Gamma_{t-2} + A_3\Gamma_{t-3} + \dots + A_q\Gamma_{t-q} + \Phi_t\Delta_t + R + \epsilon_t, \quad (22)$$

$$\Gamma_t = A_1\Gamma_{t-1} + A_2\Gamma_{t-2} + A_3\Gamma_{t-3} + \dots + A_q\Gamma_{t-q} + \Psi_t\Lambda_t + R + \epsilon_t, \quad (23)$$

$$\Gamma_t = A_1\Gamma_{t-1} + A_2\Gamma_{t-2} + A_3\Gamma_{t-3} + \dots + A_q\Gamma_{t-q} + \Phi_t\Delta_t + \Psi_t\Lambda_t + R + \epsilon_t \quad (24)$$

The equation (16) $\Gamma_t = \Phi_t \Delta_t + R(\nu^t)$ is a system of regressions or seemingly unrelated regressions (SUR). Unluckily, it turns out that Equation (16) does not fit well with the data ⁹. Such empirical incongruence is inferred to be attributed to the log-linearization of the FOCs in the derivation of (16). Accordingly, in order to overcome the local property of linear regression, we add the auto-regressive formation in the above equations.

As for the monthly data of bond yields and money growth rates, Equations (21) and (22) are applied and it is shown that the simple VAR of the interest rates, Equation (21) performs very well. On the other hand, with the quarterly data of bond yield rates, money growth rates, GDP deflator, and real GDP growth rates, all of Equations (21)-(24) are tested and it is found that Equation (22) and (24) perform better than the others.

3.2.1 Tests with Monthly data

Results from Equations (21) and (22) are displayed here. First, by running Equations (21), we obtain the impulse response functions of yield rates with respect to a shock in the federal fund rate in [Figure 3]. Within 8 months of time horizon ¹⁰, the impulse responses of all the yield rates except the long-term bond phase out.

To describe the impact of the impulse responses on the term structure of interest rates, we calculate the impulse responses with respect to bond maturities as in [Table 3] and represent them graphically in [Figure 6]. The figure shows that most of the bonds with different maturities move in the same direction but with different magnitudes or different timing, which in turn implies that with proper time lags the monetary authority can manipulate the term structure of interest rates by controlling the money growth rates continuously.

Next, we discuss the results from running Equation (22), which differs from Equation (21) in that it measures the effect of monetary policy from the stance of the quantity easing rather than interest rate policy. [Table 1], [Figure 1], and [Figure 4] confirm heterogenous responses of interest

⁹We ran regressions equation by equation. With Equation (16), in principle SUR seems more appropriate. However, in practice it usually performs better to run individual regressions with Newey-West estimate for standard deviations.

¹⁰8 months is too narrow to watch the dying-out cycles of shocks under the current system. However, we draw the impulse response functions in the very short time horizon because the feature we expect to obtain from the monthly data can be observed in 8 periods. Another pattern of dynamism, which may be present in a longer time horizon would be detected in the tests with the quarterly data.

rates with respect to money shocks.

One notable point in the analysis of monthly data is that the short-term liquidity effect is not detected, which is in contrast with the analysis of the quarterly data to follow in the next section. Probably the size of observation window matters. However, the main focus of our analysis is not for verifying or refuting the liquidity effect. Rather we are interest in exploring the controllability of diverse interest rates by adjusting the quantity of money.

3.2.2 Test with Quarterly data

Results from Equations (22) and (24) are displayed here. First, running Equations (22) reveals the impulse response functions of yield rates with respect to money shocks in [Figure 2]. Within 8 quarters of time horizon, the impulse responses of all the yield rates phase out quickly with the short-term liquidity effects between the second and fourth quarters after the arrival of the shock.

To describe the impact of the impulse responses on the term structure of interest rates, we calculate the impulse responses with respect to bond maturities as in [Table 2] and represent them graphically in [Figure 5]. The figure shows that all of the bonds with different maturities move in the same direction but with different magnitudes, which in turn implies that with proper time lags the monetary authority can manipulate the term structure of interest rates by controlling the money growth rates continuously.

On the other hand, the results from running Equation (24), which differs from Equation (22) in that it includes the history of the real GDP as a source of exogenous shocks. The impulse responses of interest rates with respect to money shocks follows the same pattern with the case of Equation (22).

4 Policy Implications

From the previous sections, it is demonstrated theoretically and empirically that the impulse response functions of the yield rates with respect to money shocks determine the shape of the term structure of interest rates. Using this property, the monetary authority can implement a certain shape of the term structure of interest rates unless there is no exogenous stochastic shocks. Thus, what the monetary authority has to concern about are the representability of the term structure as well as the time lags of its implementation.

The representability and the time lags of the implementation process are determined by the dimensions of the matrices ($\{A_i, i = 1, 2, \dots, p\}$ and Φ) as below.

$$\Gamma_t = A_0 + \sum_{i=1}^p A_i \Gamma_{t-i} + \Phi \Delta_t + \epsilon_t \quad (25)$$

An easier criterion for the representability and the time lags of the implementation process is to check an impulse response matrix, which is defined to be a stack of impulse response function values with respect to maturities and time horizon. Define the impulse response matrix Ξ to be a $q \times T$ matrix, where q is the time horizon to take before all the impulse responses completely phase out and T is the types of bond maturities available in the market. If $q \leq T$, then the representability of the system is limited to $\dim(\Xi) \leq q$. If $q > T$ and $\dim(\Xi) > T$, then the composite effect of the money growth rates during the last q quarters can represent any arbitrary term structure of interest rates. Thus, at least the horizon of impulse response functions should be longer than the kinds of assets available in the market in order to guarantee the representability. The time lags of implementation, is not easy to answer due to the presence of multiple solutions. However, the higher dimension of Ξ is more likely to raise the likelihood of attaining at a certain term structure of interest rates within a shorter time horizon.

4.1 Optimal monetary policy

Taylor rule is based on the inflation rate and the GDP gap as below. It explains the US monetary policy during 1980s and 1990s very well. However, since 2001, the federal fund rate has stayed far below the prescribed level by Taylor rule, which is due to the sluggish recovery of labor market. Of course, the GDP gap is closely related to the unemployment rate but it is not a perfect substitute.

$$\begin{aligned} i_t &= 0.04 + 1.5(\pi_t - 0.02) + 0.5(y_t - y_t^p) \\ i_t &= \bar{i} + \phi_\pi(\pi_t - \bar{\pi}) + \phi_x(y_t - y_t^p) \end{aligned}$$

Accordingly, in order to enhance the plausibility, the models of optimal monetary policy following the tradition of Taylor rules have evolved in the directions of adopting the auto-regressive features to inflation rate and GDP gap (Woodford(2003)).

In this section we suggest a criterion based on the term structure of interest rate and the past history of inflation rates and GDP gaps as an alternative.

$$\begin{aligned} \Gamma_t &= B_0 + B_1 \Pi_t + B_2 X_{t-i} + \epsilon_t \\ \Pi_t &= \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \dots \\ \pi_{t-n+1} \\ \pi_{t-m} \end{bmatrix}, X_t = \begin{bmatrix} y_t - \bar{y}_t \\ y_{t-1} - \bar{y}_{t-1} \\ \dots \\ y_{t-n+1} - \bar{y}_{t-n+1} \\ y_{t-m} - \bar{y}_{t-m} \end{bmatrix} \end{aligned} \quad (26)$$

Equation (26) is derived from introducing the delayed responses of monopolistic producers to a setup by Dixit and Stiglitz(1977). The legitimacy of (26) does not consist in that it integrates the existing models but in that it can be derived from solving a Ramsey setup. In the context of Ramsey, a search for an optimal monetary policy is defined as follows. Suppose a Robison Crusoe model with adjustment time lags. Then the optimal monetary policy function stems from the next maximization program of the monetary authority:

$$\begin{aligned} V^* &= \max_{\Delta_t} V(\Pi_t(\Delta_t), \Gamma_t(\Delta_t), X_t(\Delta_t), \Delta_t) \\ &\text{subject to} \\ \Gamma_t &= \Phi_t \Delta_t + R(\nu^t), \end{aligned} \quad (27)$$

the budget constraint of which comes from the consumer's maximization program in the earlier section. Then the optimal money growth rate function (Δ_t) should rely on Π_t, Γ_t, X_t ¹¹. Thus, in turn the optimal term structure of interest rates defined by the optimal monetary policy looks like (26).

4.2 Path dependency of interest rates

Path dependency is the case in which past activities affect the progress of the future events as well as determine the current state of an economy. The determination of the term structure of interest

¹¹ $\Gamma_t(\Delta_t)$ is derived from the consumer's program and is equivalent with the budget constrain here. However, in order to obtain exact forms of Π_t and X_t in the functions of Δ_t , LM condition (or money equation (8)) and an aggregate supply condition should be added.

rates so far discussed has demonstrated the feature of path dependency. Accordingly, an optimal monetary policy should depend on the past history of inflation rate and GDP gap.

Suppose that markets are incomplete and the monetary authority purchases government issued bond in return for new money. Then, the pattern of open market operation brings about the differential effect of monetary policy and the heterogeneous policy time lag. In other words, depending on what maturity of bonds are purchased or sold in the market by the monetary authority, the impulse response over time and the time required to reach a certain shape of the term structure of interest rates may differ. In addition, there exist various ways of reaching a certain shape of the term structure of interest rates with the same or different time lags.

5 Concluding Remarks

Our paper explores a transmission mechanism of monetary policy through bond market. Based on the assumption of delayed responses of economic agents to monetary shocks, we derive a system of equations relating the term structure of interest rates with the past history of money growth. The equations are empirically tested with the US data after being modified to a VAR setup. Such a modification process is inevitable to avoid the locality resulting from log linearization of the first order conditions. Impulse response functions of various yield rates with respect to monetary shocks as well as to the short-term interest rate (such as federal fund rate in the US) reveal that the reactions of the yield rates may vary across the bonds with different maturities in terms of directions as well as magnitudes. From this observation, we find out that the policy of maintaining a certain level of the short-term interest rate may lead to different economic consequences depending on the differences of the past monetary policy. Such path-dependency of monetary policy induces that monetary policy targeting a certain shape of the term structure of interest rates could be implemented only with a certain time lags.

So far the effects from other omitted exogenous variables are neglected. For example, seemingly significant parameters or variables, such as the variability of money velocity and a shift in consumers' preference, are not fully considered. Such an omission problem would be in practice not a big deal only if omitted variables are deterministic. On the contrary, when the omitted variables are stochastic and are not observable, an algorithm for implementing an optimal monetary policy becomes a usual Kalman filtering setup. This issue is very critical at the stage of application and

is expected to be dealt in the following works.

References

- [1] Alvarez, F., R. Lucas, and W. Weber, 2001, Interest rates and inflation, *American Economic Review* 91, 219-225.
- [2] Bernanke, B. and J. Boivin, 2003, Measuring the effects of monetary policy: a factor-augmented vector auto-regressive(FAVAR) approach, FRB working paper.
- [3] Bils, M, P. Klenow, and O. Kryvsov, 2003, Sticky prices and money policy shocks, FRB Minneapolis Quarterly Review 27, 2-9.
- [4] Bordo, M. and Haubrich, 2004, The yield curve, recessions and the credibility of the monetary regime: long run evidence 1875-1997, *NBER working paper* 10431.
- [5] Clarida, R., J. Gali, and M. Gertler, 1999, The science of monetary policy: a new Keynesian Perspective, *Journal of Economic Literature* 37, 1661-1707.
- [6] Clarida, R., J. Gali, and M. Gertler, 2000, Monetary policy rules and macroeconomic stability: evidence and some theory, *Quarterly Journal of Economic Literature* 115, 147-180.
- [7] Dixit, A. and J. Stiglitz, 1977, Monopolistic competition and optimum product diversity, *American Economic Review* 67, 297-308.
- [8] Eichenbaum, M. and J. Fisher, 2004, Fiscal policy in the aftermath of 9/11, *NBER working paper* 10430.
- [9] Hendry, S., W. Ho., and K. Moran, 2003, Simple monetary policy rules in an open economy, limited participation model, Bank of Canada working paper 2003-38.
- [10] Hong, K., 1997, Fluctuations in consumer durables expenditure and fixed investment in Korea, *International Economic Journal*, 1997.
- [11] Hong, K., 1996, A comment on durable goods consumption, *Journal of Monetary Economics*, 1996
- [12] Lucas, R., 2000, Inflation and welfare, *Econometrica* 68, 247-274.

- [13] Lucas, R. and N. Stokey, 1987, Money and interest in a cash-in-advance economy, *Econometrica* 55, 491-513.
- [14] Monnet, C and W. Weber, 2001, Money and interest rates, FRB Minneapolis Quarterly Review 25, 2-11.
- [15] Orphanides, A., 2003, Monetary policy in deflation: the liquidity trap in history and practice, FRB working paper.
- [16] Taylor, J., 1993, Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy* 39, 195-214.
- [17] Thomas, J., 2002, Is lumpy investment relevant for the business cycle?, FRB Minneapolis research department staff report 302
- [18] Woodford, M., 2003, *Inflation and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.
- [19] Yun, T., 1996, Nominal rigidity, money supply endogeneity, and business cycles, *Journal of Monetary Economics* 37, 345-370.

Appendix

[Table 1] Orthogonal Impulse Response Functions of Yield Rates to Money Shocks (Monthly Data)

Period	FF	TB3mon	TB6mon	TB1yr	LGTB
0	2.36771	1.41809	1.9391	1.13211	-0.20733
1	6.14088	5.92639	5.81499	4.22015	2.22498
2	7.27664	8.21618	7.56199	5.8756	3.6768
3	8.87741	10.0781	8.89956	6.90694	4.02123
4	9.39684	9.94298	8.74984	6.62403	4.54423
5	8.3166	7.29077	6.55254	5.43528	4.53766
6	8.02364	6.3077	5.6971	5.08671	4.37086

[Table 2] Orthogonal Impulse Response Function of Yield Rates to Money Shocks (Quarterly Data)

Period	FF	TB3	TB6	TB1yr	LGTB
0	167.245	153.302	157.231	160.383	211.098
1	0.246421	-0.19952	0.16491	0.510467	0.798844
2	-65.7299	60.4593	-61.7011	-62.7765	-83.1652
3	19.6155	18.2779	18.531	18.6959	23.7394
4	15.0446	13.8131	13.9465	14.0503	17.5664
5	-6.86679	-6.64232	-6.89767	-7.15451	-10.4128
6	-0.4169	-0.64344	-0.75822	-1.00428	-2.19787

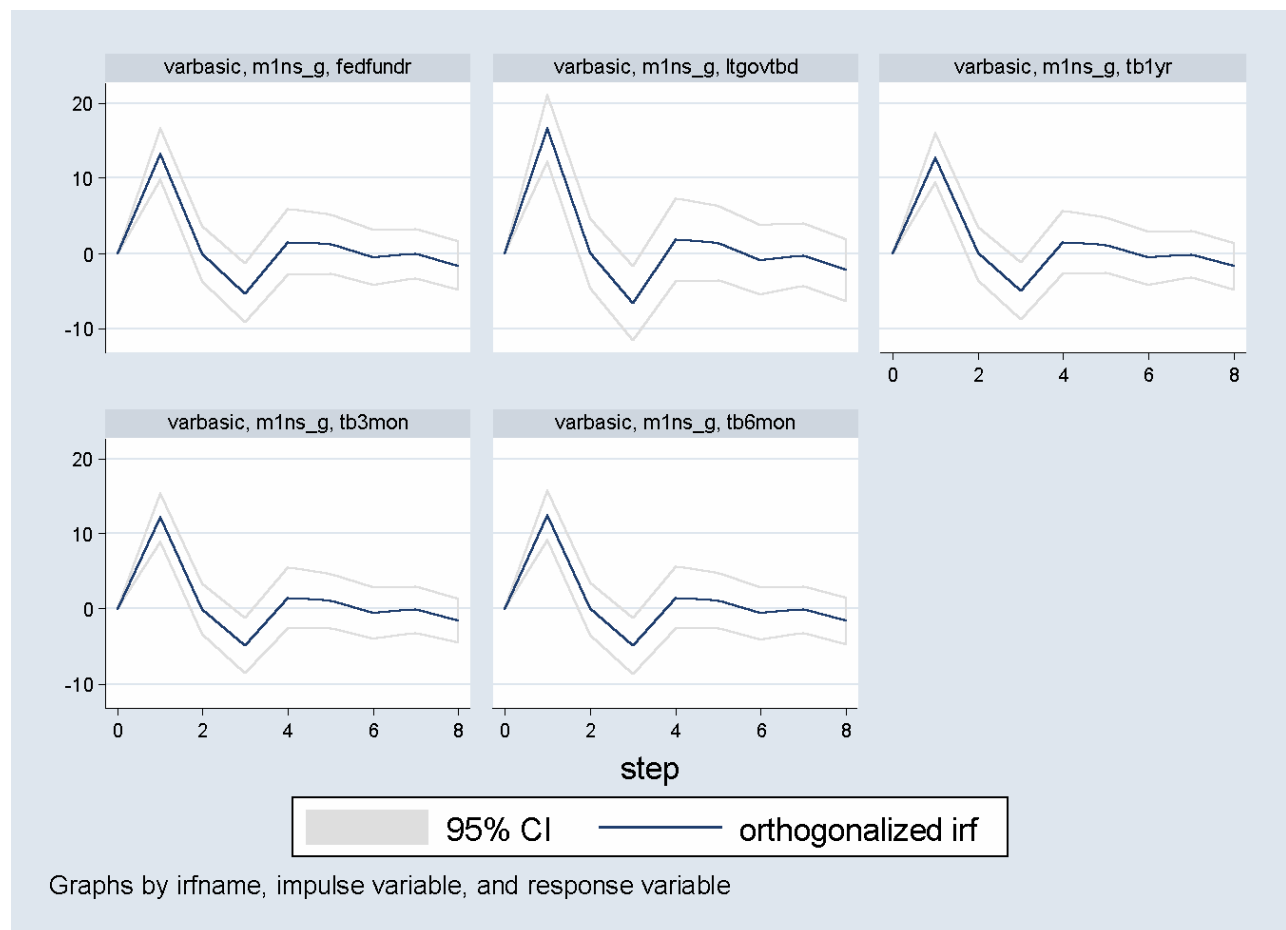
[Table 3] Orthogonal Impulse Response Function of Yield Rates to Federal Fund Rate Shocks (Monthly Data)

Period	FFR	TB3	TB6	TB1yr	LGTB
0	0.491611	0.3121	0.289359	0.239936	0.080028
1	0.698939	0.493495	0.436963	0.356228	0.121261
2	0.692776	0.491415	0.445463	0.375442	0.129747
3	0.637012	0.425693	0.391827	0.334656	0.125267
4	0.555573	0.391656	0.360064	0.309433	0.13618
5	0.516552	0.382242	0.359379	0.317069	0.157767
6	0.503729	0.373448	0.357549	0.317967	0.170279
7	0.492744	0.358745	0.345734	0.308132	0.170361

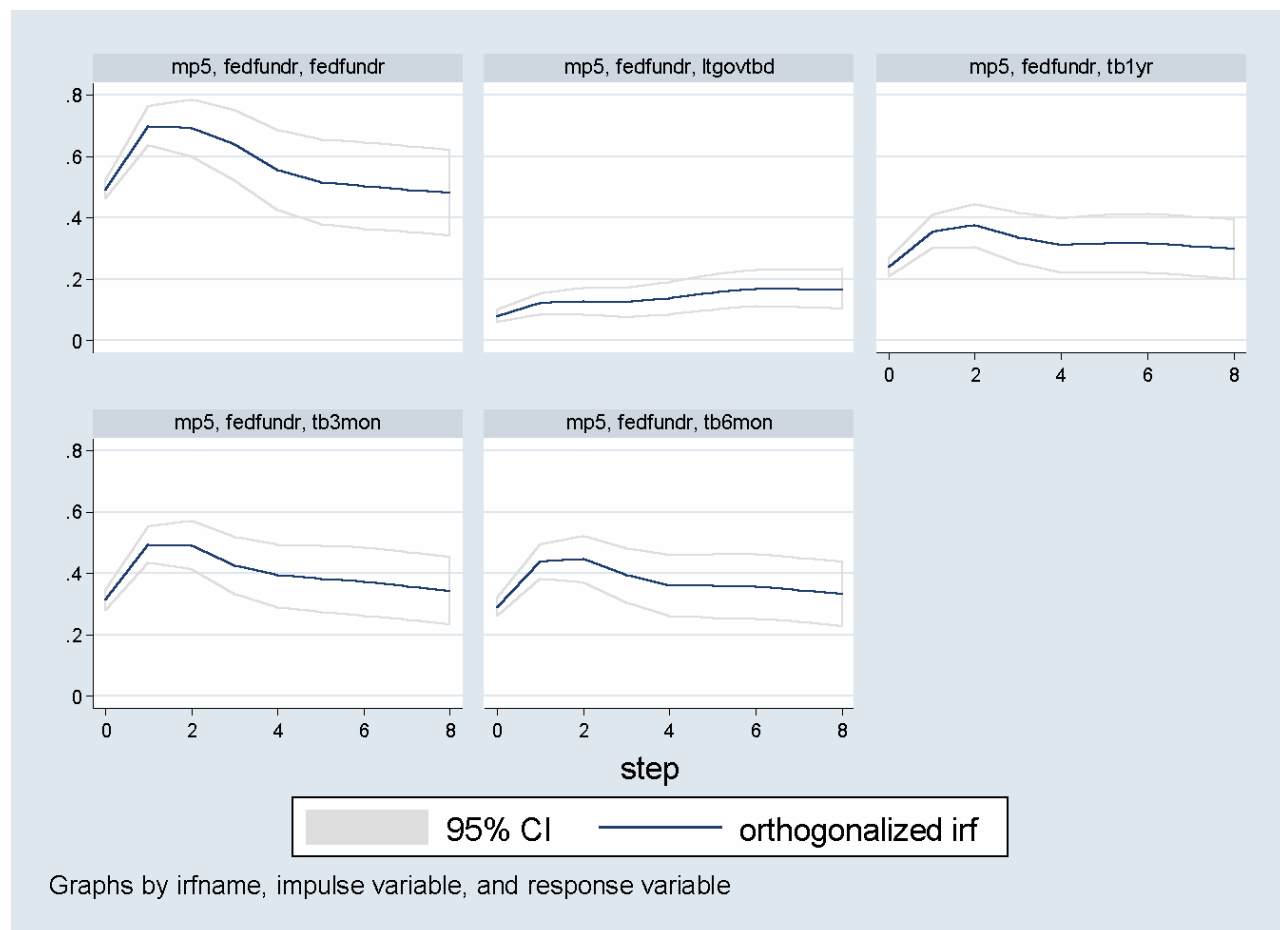
[Figure 1] Orthogonalized Impulse Response Functions with Exogenous Money Shocks (Monthly Data)



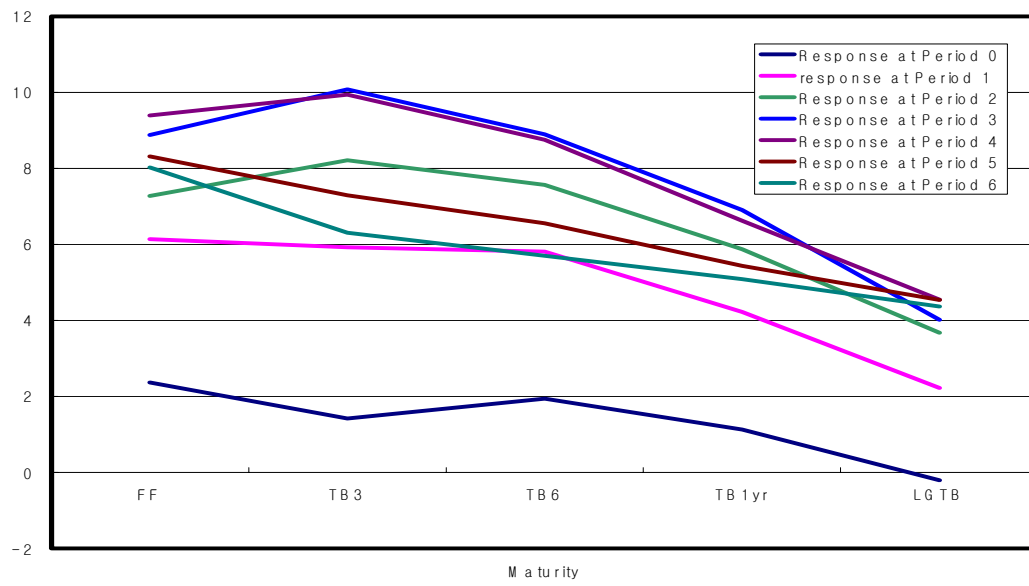
[Figure 2] Orthogonalized Impulse Response Functions with Exogenous Money Shocks (Quarterly Data)



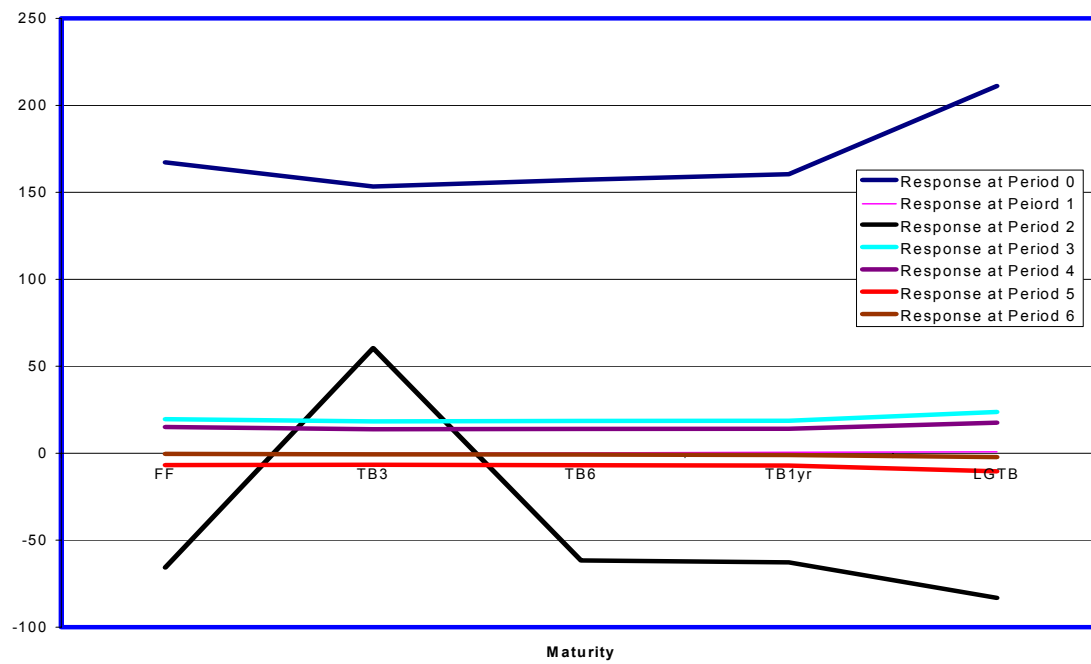
[Figure 3] Orthogonalized Impulse Response Functions with Federal Fund Rate Shocks (Monthly Data)



[Figure 4] Impulse Response of the Term Structure of Interest Rates to monetary shocks-Monthly



[Figure 5] Impulse Response of the Term Structure of Interest Rates to monetary shocks-Quarterly



[Figure 6] Impulse Response of the Term Structure of Interest Rates to FFR shocks (Monthly Data)

