A General Equilibrium Analysis of Land Use Restrictions and Residential Welfare ⋆

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Abstract

We consider the general equilibrium implications of land use restrictions which result in a reduction of otherwise profitable residential development. If the regulations affect a significant amount of land, they may have important effects on the rest of the regional economy - increasing rents and densities on lands not subject to the regulation, causing the conversion of lands from alternative uses, increasing the net developed area in the region, and decreasing consumer welfare. We develop a flexible general equilibrium simulation of the economic effects of land use restrictions, explicitly considering the distributional effects upon owners of different types of land and upon housing consumers. The results of our simulation show that the most significant economic effects of land use regulations occur outside of the designated area. The prices and rents of non-restricted lands increase significantly, and the well being of housing consumers is further affected through these linkages.

1 Introduction

In this paper we consider the consequences of government regulations that reduce the amount of land that would otherwise be available for residential development. We consider those circumstances in which the government designates a part of the urban region as "open space." This designation may be motivated by aesthetic considerations, as when the land designated will enhance the views of local residents or the general amenity of the area. Alternatively, the designation may be motivated by environmental ecological concerns, as when specific land is declared to be "critical habitat" for some protected species of plant or wildlife. We do not consider the specific benefit side of the designation, but we analyze the economic consequences of the designation for the land and housing markets. If the amount of developable land regulated is "small," then in a static equilibrium, these economic impacts can be calculated in a straight forward manner. A designation will decrease the value of the land if it prohibits the most economically beneficial uses of the land. Absent external effects, the costs can be measured simply as the change in the market value of the designated lands.

On the other hand, if the amount of regulated land is at all "large," the economic impacts are more complicated. When large amounts of developable land are regulated, the economic effects of the regulation may no longer be confined to the lands that have been designated. In these instances, the price of land and the pattern of land usage throughout the region will adjust to reflect the scarcity of developable land induced by the regulation. In these circumstances, a more general approach is needed to trace through the impacts.

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Calculating the impacts from regulating non-negligible quantities of land requires an explicit model of the interrelationships of consumers and producers in the economy. Understanding the economic consequences of land use restrictions also necessitates explicitly considering the counterfactual: the land use decisions that would have occurred but for the imposed regulations. This paper provides a general equilibrium framework in which the impacts of land use restrictions can be analyzed in a systematic manner. We consider the economic implications of restricting raw land that would otherwise have been used to produce housing in a region whose population base is expected to expand.

We consider a closed region whose economic base is given, where mobility within the region is costless, but mobility to other regions is prohibitively expensive. In the alternative, "open region" formulation, where mobility between regions is costless, the well being of the region's residents is determined exogenously. Thus, the competitive equilibrium must yield the same level of utility for residents regardless of regulation in the region. This implies that the entire cost of regulation is reflected in the change in market value of the regulated lands, as interregional migration equilibrates consumer utility.

We examine the impacts of land use regulation in a closed region through its effects upon the well being of producers and consumers of housing within the region. Changes arise because some significant amount of land cannot be used as intensively in producing housing after regulation. "Significant" means enough land to alter the supply of land available for residential development. In a stylized model of the regional economy, we evaluate the impacts of these regulations on the spatial allocation of capital, on the density of housing development, and on housing and land prices throughout the region. We also analyze the net effect of the land designation on the well-being of households and the distribution of rents among the region's landowners.

Section 2 below surveys the sparse and incomplete literature on this issue and summarizes prior work by economists studying environmental regulation of land uses. Our formal model of land use restrictions in the regional economy is sketched out in Section 3. Then, in Section 4 we trace out the most important impacts of land use restrictions using this model to deduce the qualitative effects. Section 5 presents a quantitative application. We use the model to estimate the magnitude of the economic consequences of land use regulation using stylized but reasonable parameters reflecting a regional economy. Section 6 extends the model to examine the most extreme example of land use restriction in which the designation completely surrounds the region. This is analogous to an urban growth boundary and shows the generalizability of our analysis. Section 7 provides a brief conclusion.

2 Prior Research

The empirical literature linking land use regulation to housing outcomes is extensive, but much of it consists of case studies. Peng and Wheaton [1994] analyze the monopoly mechanisms by which land is supplied for residential development in Hong Kong, concluding that the institution results in higher housing prices and more capital intensive development, but little reduction in aggregate supply. Hannah et al. [1993] analyzed five development projects in Seoul, Korea, linking government land use restrictions to land prices and the high internal rates of return to apartment developers during the 1980s.

There is a more extensive economic literature examining impacts of fees, rather than prohibition on development, on developers and consumers. For instance, Watkins [1999] develops a framework for analyzing the impacts of development charges, and he shows how the fee will be shared between landowners and home buyers. Singell and Lillydahl [1990] undertake an empirical examination of impact fees charged to all newly constructed houses. They discover that existing home prices may rise in the presence of impact fees for new homes. They hypothesize that this is because home buyers predict that property taxes on existing homes will decrease to offset the increased tax revenue generated by the impact fees. Skidmore and Peddle [1998] estimate the effects of development impact fees imposed in some municipalities in a single Illinois county. They find that residential development rates were reduced by 25 percent in fee areas compared to similar municipalities that did not enact the development fees. A recent paper by Quigley and Rosenthal [2004] presents a survey of empirical evidence on the link between land use regulation and housing prices, finding important effects of land use regulation on housing prices.

Another recent paper by Kiel [2004] reviews the economic literature on the effects of one form of land use regulation, environmental restrictions on the housing market. After a comprehensive review of regulations under the Clean Air Act, the Safe Drinking Water Act, the Environmental Policy Act, and the Coastal Zone Management Act, among others, she concludes, "It is surprising how little is known about the impacts of environmental regulations on the price and quantity of housing in the United States." (page 20) Kiel notes that if environmental regulation "removes a significant amount of land from possible development, then the price of remaining developable land should increase, thus increasing the cost of supplying housing. . . ." However, she develops no formal model of the land market, and she presents no empirical evidence at all on the issue.

There is some theoretical work on this topic. Brueckner [1990] has developed a model of growth controls in an open city. He shows that while consumers may remain indifferent about growth controls (as a consequence of the open city assumption), landowners may gain or lose when the growth controls affect land prices in the region. The model shows that growth controls will increase the value of developable land, while decreasing the value of land rendered undevelopable by the restrictions. More recently, Lee and Fujita [1997] developed a theoretical model of efficient greenbelt location within an urban area. Their analysis emphasizes the local public good amenities provided by the greenbelt, rather than the economic effects of the restrictions on land supply.

In the next section we develop a theoretical model of land use in a regional economy. The model is static in the sense that it focuses on the equilibrium of the local economy. Using this model, we then examine the impacts of land supply restrictions. We evaluate the impacts on both landowners and consumers in the region-wide economy rather than merely on the designated lands. In our model, the regulation increases the demand for housing services elsewhere in the region by reducing the availability of land to produce housing in the regulated areas. This increased demand causes housing and land prices to increase and residential densities to increase as well. In these instances this decreases the well being of consumers of housing and increases the welfare of landowners and current owner occupants. The contribution of this work is to analyze the importance of these region-wide effects of regulation in evaluating the economic impacts of land use restrictions. As our results indicate, in many instances these economy-wide effects are much more important than the effects which arise in the regulated areas.

3 The Basic Economic Model

This section describes the basic economic model used to determine the initial economic equilibrium. Models of this kind were first introduced by Alonso [1964], and extended and popularized by Mills [1967], Muth [1969], and Beckmann [1969]. Papers by Wheaton [1974] and Pines and Sadka [1986] are among the better known examples of attempts to deconstruct these models using comparative statics. Brueckner [1987] provides a comprehensive review of "Muth–Mills" literature. We follow Brueckner's model closely, adopting his notation as far as possible. We then expand the model to investigate the removal of developable land by land use restrictions to examine the effects of these exogenous regulations on the equilibrium of the region.

Consider a geographic region region with N identical consumers of income y, whose well-being depends on their consumption of housing, q, at price, p, and a numeraire good, c. At any chosen location, x, measured as the distance to the central place of employment, residents must pay commuting costs $t(x)$. For convenience only, we assume transportation costs are linear, tx.

Consider the consumer's problem. The consumer acts to maximize a well behaved utility function

$$U(c, q). \tag{1}$$

Housing (q) is a scalar indexing the quantity of housing consumed. The budget constraint facing the consumer,

$$y = c + p(x)q(x) + tx, \tag{2}$$

reflects the fact that income net of transportation costs is spent on housing and other goods. [1]

Consumers choose a location and quantity of housing to consume at price $p(x)$. These two choices determine the total commute costs, and also the residual income to be spent on the numeraire good. In equilibrium, the marginal rate of substitution between housing and the numeraire good must equal the ratio of their prices, where subscripts refer to partial derivatives.

$$max\ U(y - p(x)q(x) - tx, q(x)) \ \Rightarrow \ \frac{U_2(y - p(x)q(x) - tx, q(x))}{U_1(y - p(x)q(x) - tx, q(x))} = p(x). \tag{3}$$

Since all consumers are assumed to be identical, everyone must have a common

[1] Note that, in contrast to Lee and Fujita [1997], we do not consider the possibility that the designation itself creates amenities which increase consumer welfare.

level of utility, $\overline{u}$.

$$max\ U(y - p(x)q(x) - tx, q(x)) = \overline{u}. \tag{4}$$

In equilibrium, consumers will have the same utility level regardless of where they locate within the region. Consequently, the price and quantity of housing consumed must vary systematically by location within the region. The schedules of prices and quantities at all locations are determined by the solution to equations (3) and (4).

Now consider production and the supply of housing at all locations. We assume that developers are price-takers in the markets for land and capital, and that the cost of capital is constant throughout the region, while the price of housing and land varies spatially. Let $K(x)$ represent the amount of capital, and $L(x)$ represent the amount of land used in the production of housing at location x. Assume a housing production function, $H(K, L)$, characterized by constant returns to scale and concavity in input substitution. With a production technology exhibiting constant returns to scale, the problem faced by firms producing housing can be simplified. Each producer chooses only a capital intensity, $S(x) = K(x)/L(x)$.

$$H(K, L) = L \cdot H(K/L, 1) = L \cdot h(S(x)). \tag{5}$$

Given the housing price at any location, profit at any location can be written as

$$\pi = pLh(S) - iSL - r(x)L = L[ph(S) - iS - r(x)], \tag{6}$$

where i is the spatially invariant price of capital and $r(x)$ the price of land at location, x. In equilibrium, the marginal revenue product is equal to the marginal cost of each input, and competitive producers will earn zero profits. Equation (7) determines the capital-to-land intensity for profit maximizing producers while equation (8) represents the zero-profit condition for the competitive producers. Taken in concert, equations (7) and (8) fully characterize the production side of the model. The capital intensity of housing and the price of land vary systematically by location within the region.

$$p(x)h'(S(x)) = i, \tag{7}$$

$$p(x)h(S(x)) - iS(x) = r(x). \tag{8}$$

The region must also achieve an economic equilibrium in two other senses. First, land must be successfully bid away from its alternative use. Let r_a represent the opportunity cost of land, and $\overline{x}$ be the distance to the border of the economically productive region. Then the rent for land devoted to housing at the border must equal the rent in its highest alternative use,

$$r(\overline{x}) = r_a. \tag{9}$$

It can easily be shown that $\frac{\partial r}{\partial x} < 0$. Thus, equation (9) specifies that all land devoted to housing is successfully bid away from the alternate use.

Secondly, the supply of housing must equal the demand for housing within the region as a whole. Without loss of generality, assume that each household contains one individual, and recall that $h(S(x))$ is the quantity of housing produced, while $q(x)$ is the quantity of housing demanded by the representative consumer at any location. Then, $2\pi x \; dx \cdot h(S(x))/q(x)$ yields the number of households at distance x, and integration over the entire region, a circle of radius $\overline{x}$, yields the population of the region, N.

$$\int_0^{\overline{x}} 2\pi x \frac{h(S(x))}{q(x)} dx = N. \tag{10}$$

The economy is fully characterized by six equations, two equations representing consumer choice, subject to the constraint that all consumers must have the same utility, two equations representing the housing production sector, subject to the constraint that all producers earn identical normal profits, and the two equations representing the spatial equilibrium in the region — equations (3), (4), (7), (8), (9), and (10). These six equations have six unknowns: four functions, $p(x)$, $r(x)$, $s(x)$, $q(x)$, and two constants, $\overline{x}$, and $\overline{u}$. The values which solve this system describe the spatial pattern of housing prices, land rents, capital intensity of housing, and housing density. The model also solves for the physical size of the built up region and the common level of utility of the residents. Simultaneously solving these equations determines the equilibrium of the region. The N residents of the region all obtain constant utility of $\overline{u}$. Housing prices decline with distance from the center. Land prices decline more steeply than housing prices. Population density decreases with distance from the center.

In the following section, we extend the model to allow for the removal of some lands from the land supply. We then use comparative statics analysis to examine the changes to the model from the exogenous constraints placed on land use in the region.

4 Impacts of Land Use Restrictions

Suppose that a regulator chooses to restrict development on certain lands in this regional economy. As noted earlier a general equilibrium approach is needed when the designation alters the behavior of surrounding land owners in addition to the owners of regulated lands. The regulation is characterized by its size and location within the region, both of which play important roles in determining the ultimate impacts on the economy. In this paper we assume land use regulations can be approximated by three parameters, the distance, x^*, between the center of the region and the regulated area, the number of radians that the regulation occupies, k, and the "depth" of the restrictions,

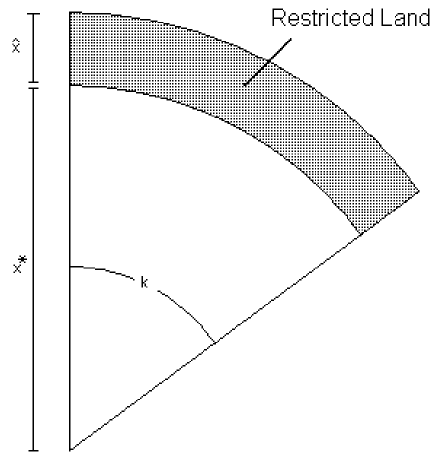


Figure 1. Geography of Land Use Restriction

$\hat{x}$. That is, we assume the restrictions affect k radians of an annulus of width $\hat{x}$ at distance x^* from the center of the region. Given this formulation of the regulation, the total restrcited area is $k\hat{x}(x^* + \frac{\hat{x}}{2})$. See Figure 1 for a schematic of the regulated area.

If $x^* > \overline{x}$, then the land use restrictions are located outside of the urbanized region. The economy within the built up region is not affected, and the impact of the regulations are simply the lost land value of the regulated lands. If r_d represents the rents of regulated lands, then the economic costs are given in equation (11).

$$Costs = k\hat{x}(x^* + \frac{\hat{x}}{2})(r_a - r_d) \tag{11}$$

Equation (11) shows that if the regulation does not reduce the rent of the regulated land ($r_d = r_a$), then the regulation is costless.

We now turn to the case in which there are impacts upon consumers and producers outside of the regulated lands. This occurs when restrictions are placed on land that would otherwise be used in the production of housing services (i. e., when $x^* < \overline{x}$). In this case, the regional equilibrium must adjust to accommodate the households who would otherwise reside in the restricted lands. For simplicity, we assume that restricted lands cannot be used to produce housing at all. We also assume that the depth of the regulation is such that it is not optimal for residents to "leap-frog" and develop beyond the restricted lands. This assumption is merely for analytic tractability, and can be relaxed in practice. [2] However, we do allow builders to expand the city by converting land to housing at the boundary. [3] This pattern of regulation can be reflected in the economic model by modifying equation (10). Equation (10) becomes

$$\int_0^{x^*} 2\pi x \frac{h(S(x))}{q(x)} dx + \int_{x^*}^{\overline{x}} (2\pi - k) x \frac{h(S(x))}{q(x)} dx = N. \tag{12}$$

Equation (12) states that the population must fit within the built up region, which only includes $2\pi - k$ radians past a distance of x^*. It reinforces the notion that land use restrictions will have only minor impacts if it is located outside of the built up region (when $x^* = \overline{x}$, (12) reverts to (10), as it does if $k = 0$). The new equilibrium will be determined by simultaneously solving the new system of six equations, (3), (4), (7), (8), (9) and (12).

We rely upon comparative statics to compare the equilibrium of the economy with and without the land use restrictions. The comparative statics solution for the system of equations representing the model is reported in Appendix A, but the intuition is straight forward, and the important results can be summarized. In the model, restricting the use of land reduces the supply of land available for housing. For the system to remain in equilibrium, the displaced residents must find housing elsewhere. The increased demand for housing elsewhere causes unregulated lands to be developed more intensely, including the development of lands at the periphery that would not have been developed but for the regulation.

The parameter k measures the radians of restricted land. Increasing k, while

[2] Note: Although we also assume that restricted lands are prohibited from producing housing, the results are qualitatively identical if the regulation merely decreases the density of housing. The model can easily incorporate "costly delay" in the development process. This possibility is formally the same as an excise tax on housing, but does not qualitatively change the results.

[3] Of course, if $k = 2\pi$ the regulation is analogous to the designation of an urban growth boundary. See Section 6 for the impacts of a growth boundary.

holding x^* constant, increases the amount of regulated land. When $k = 2\pi$, restricted lands fully surround the region, and the regulation becomes a growth boundary. As indicated in (13), an increase in k will cause housing prices to rise and the quantity of housing demanded by each consumer to fall, both of which reduce consumer utility. Housing densities increase, land prices increase, causing the distance to the rural-urban boundary to increase.

$$\frac{\partial p}{\partial k} > 0,\ \frac{\partial q}{\partial k} < 0,\ \frac{\partial S}{\partial k} > 0,\ \frac{\partial r}{\partial k} > 0,\ \frac{\partial \overline{u}}{\partial k} < 0,\ \frac{\partial \overline{x}}{\partial k} > 0. \tag{13}$$

Since x^* measures the distance from the restricted lands to the city center, increasing x^* represents locating the restricted area farther from the center of the region. As intuition suggests, this reduces the impacts of regulation. Equation (14) shows that increasing x^* decreases the price of housing, increases the demand for housing, decreases residential density, decreases land rents, increases consumer welfare, and reduces the distance to the urban-rural boundary.

$$\frac{\partial p}{\partial x^*} < 0,\ \frac{\partial q}{\partial x^*} > 0,\ \frac{\partial S}{\partial x^*} < 0,\ \frac{\partial r}{\partial x^*} < 0,\ \frac{\partial \overline{u}}{\partial x^*} > 0,\ \frac{\partial \overline{x}}{\partial x^*} < 0. \tag{14}$$

Figure 2 shows a schematic of the city shape and size with and without land use restrictions. Without regulation, the equilibrium is a circular built-up region of radius $\overline{x}_0$, represented by the broken line. The equilibrium with land use restrictions causes the conversion of lands from the alternate use to housing production, expanding the built-up region to the solid line at $\overline{x}_1$. Along the ray from the center of the region to point **A**, the price of all lands has increased (As shown in Figure 3. A). Lands that had been previously developed are worth more due to the increased scarcity of lands, causing the conversion of lands which would otherwise remain undeveloped to housing production. Meanwhile, Figure 3. B, a cross section of land prices from the center of the region through

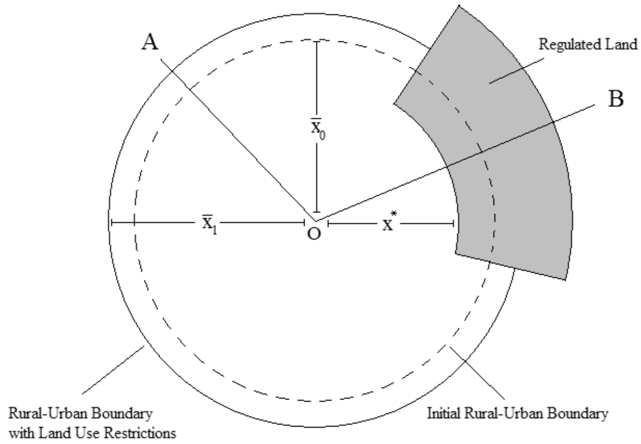
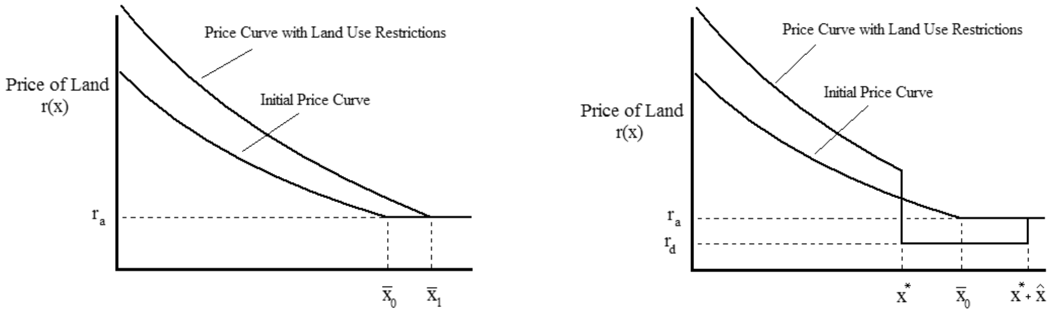


Figure 2. Equilibrium with Land Use Restrictions



3.A Origin to Point A　　　　3.B Origin to Point B

Figure 3. Equilibrium Land Price Gradients

the restricted lands to point **B**, again shows that lands not regulated are more valuable, while the restricted lands are less valuable. Specifically, for $x^* < x < \overline{x}_0$, lands that would have been developed to produce housing now simply earn r_d, and lands located $\overline{x}_0 < x < x^* + \hat{x}$ have price r_d instead of r_a.

4.1 Effects upon Landowners

Owners of unregulated lands will benefit from increased land prices, while landowners of restricted land stand to lose from the designation. Are the gains to the winners greater than the losses of the losers? To answer this, consider the aggregate land rent in the region, R.

$$R = \int_0^{x^*} 2\pi x \cdot r(x, x^*, k)\ dx + \int_{x^*}^{\overline{x}(x^*,k)} (2\pi - k)x \cdot r(x, x^*, k)\ dx. \tag{15}$$

Let us examine the impacts upon R as we alter the parameters which determine the extent and location of the regulated land in the region. To answer this question, we calculate $\frac{\partial R}{\partial x^*}$ and $\frac{\partial R}{\partial k}$, using Leibniz' Rule.

$$\frac{\partial R}{\partial k} = \int_0^{x^*} 2\pi x \cdot \frac{\partial r(x)}{\partial k}\ dx\ - \int_{x^*}^{\overline{x}(x^*,k)} x \cdot r(x, x^*, k)\ dx \tag{16}$$

$$+ \int_{x^*}^{\overline{x}(x^*,k)} (2\pi - k)x \cdot \frac{\partial r(x)}{\partial k}\ dx + (2\pi - k)\overline{x}\, r(\overline{x}) \frac{\partial \overline{x}}{\partial k}$$

$$\frac{\partial R}{\partial x^*} = \int_0^{x^*} 2\pi x \cdot \frac{\partial r(x)}{\partial x^*}\ dx\ + 2\pi x^*\, r(x^*) + \int_{x^*}^{\overline{x}(x^*,k)} (2\pi - k)x \cdot \frac{\partial r(x)}{\partial x^*}\ dx \tag{17}$$

$$+ (2\pi - k)\overline{x}\, r(\overline{x}) \frac{\partial \overline{x}}{\partial k} - (2\pi - k)x^* r(x^*) \frac{\partial r(x)}{\partial x^*}$$

Without assuming functional forms for the utility of residents and the production technologies available to producers, equations (16) and (17) are of ambiguous sign. Nevertheless, it is useful to decompose the change in rents into the increases and decreases to landowners in different locations. Table 1 describes the different categories of landowners, while Figure 4 displays the geographical locations of these landowner groups. The landowners can be divided into four separate groups:

(1) The owners of land which would otherwise be developed and is not regulated, shown as region **A** in Figure 4. These landowners gain from the increased price of their land as it gets developed more intensely. The total gains for these landowners are equal to $\int_0^{x^*} 2\pi x(r_1(x) - r_0(x))dx + \int_{x^*}^{\overline{x}_0} (2\pi - k)x(r_1(x) - r_0(x))dx$.

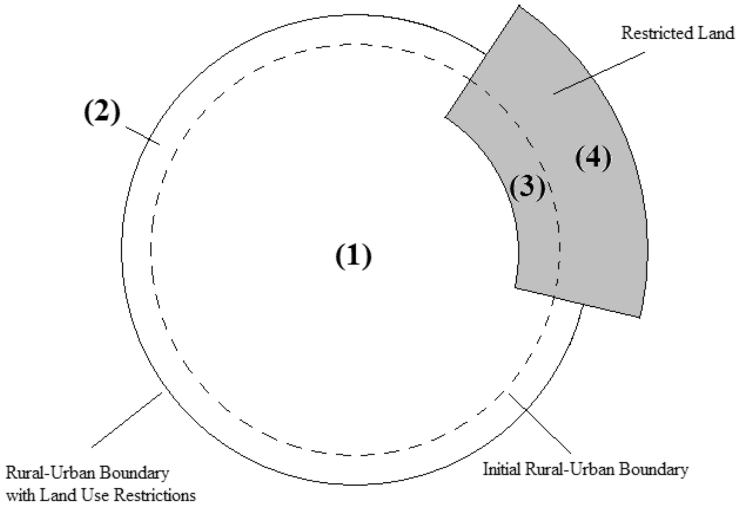


Figure 4. Geography of Land Use Restriction Impacts

(2) The owners of land which would otherwise be undeveloped at the edge of the region is now developed as a consequence of the land use restrictions. These lands are shown as region **B** in Figure 4. These owners gain increased land rents equal to $\int_{\overline{x}_0}^{\overline{x}_1}(2\pi - k)x(r_1(x) - r_a)dx$.

(3) The owners of land which would otherwise be developed but is now restricted, shown as region **C** in Figure 4. These landowners lose from not being able to develop their lands. The aggregate loss in rent is $\int_{x^*}^{\overline{x}_1} kx(r_d - r_0(x))dx$

(4) The owners of land that would otherwise remain undeveloped but is now regulated, shown as region **D** in Figure 4. These landowners lose rents if their lands are regulated, and the rent to lands so restricted is lower than the alternate use of that undeveloped land (when $r_d < r_a$). We assume that $r_d = r_a$, and thus these landowners are not affected by the regulation.

It can also be shown that the slope of the housing bid-rent curve becomes steeper as more land is restricted. Totally differentiating equation (4) with respect to p and x yields

$$[U_1 \cdot (-q)]dp + [U_1 \cdot (-t)]dx = 0 \Rightarrow \frac{dp}{dx} = \frac{-t}{q}, \tag{18}$$

Table 1: Decomposition of Changes in Land Values

	Not Regulated$_1$		Regulated$_1$
Developed$_0$	(A)$\int_0^{x^*} (r_1(x) - r_0(x))\, 2\pi x\, dx +$ $\int_{x^*}^{\bar{x}_0} (r_1(x) - r_0(x))\, (2\pi - k) x\, dx$		(C)$\int_{x^*}^{\bar{x}_0} (r_d - r_0(x))\, k\, x\, dx$
Undeveloped$_0$	Developed$_1$ (B)$\int_{\bar{x}_0}^{\bar{x}_1} (r_1(x) - r_a)\, (2\pi - k) x\, dx$	Not Developed$_1$ no change	(D)$\int_{\bar{x}_0}^{x^* + \hat{x}} (r_d - r_a)\, k\, x\, dx$

Where $_0$ implies no restrictions and $_1$ implies a regulated economy. It is helpful to imagine the restrictions as keeping x^* constant, and changing $k_0 = 0 \rightarrow 0 < k_1 \leq 2\pi$

and thus,

$$\frac{\partial \left(\frac{\partial p}{\partial x}\right)}{\partial x^*} = \frac{\partial \left(\frac{-t}{q}\right)}{\partial x^*} = \frac{t}{q^2} \cdot \frac{\partial q}{\partial x^*}, \text{ and} \tag{19}$$

$$\frac{\partial \left(\frac{\partial p}{\partial x}\right)}{\partial k} = \frac{\partial \left(\frac{-t}{q}\right)}{\partial k} = \frac{t}{q^2} \cdot \frac{\partial q}{\partial k}. \tag{20}$$

Since $\frac{\partial q}{\partial x^*}$ is positive, and both t and q are positive, equation (19) implies that the slope of the price-per-unit-of-housing function increases when the distance to the regulated land. Since the slope of this function is *negative*, this implies that the bid-rent function becomes flatter and the housing price function also becomes flatter. Thus, regulating more land (bringing the border of the regulated lands closer to the center of the region) causes the slope of the bid-rent curve to become steeper. Equation (20) reinforces the conclusion that more regulated land makes the housing price gradient steeper (since $\frac{\partial q}{\partial k}$ is negative and an increase in k implies more restricted land). Similar logic yields the same results for the land price function. [4]

4.2 Effects upon Residents

To summarize the economic effects of land use restrictions upon consumers, we calculate the equivalent variation (EV) of the policy implementation. The equivalent variation is the amount by which the income of the representative

[4] This has the striking result that some of the largest impacts of land use restrictions may actually occur far from the regulated lands.

consumer must be changed in the absence of the policy to yield the same utility level as if the policy had been implemented. A negative EV implies that the policy reduces the utility of the residents, e. g. consumers must have their incomes reduced to yield the same utility as they would receive with the policy in place. From equations (13) and (14) we see that when regulation constrains the amount of raw land available for development, consumers of housing are unambiguously worse off. The only question is by how much.

$$EV_{PE} = dy : \overline{u}(y,\ Restrictions) = \overline{u}(y + dy,\ No\ Restrictions) \tag{21}$$

In a partial equilibrium framework, consumer EV_{PE} can be calculated according to equation (22), where $\overline{u}_0$ refers to the pre-regulation equilibrium utility level, $\overline{u}_1$ refers to the equilibrium utility level after the land use restrictions, and u_y is the marginal utility of income as calculated by the utility function.

$$dy = \frac{\overline{u}_1 - \overline{u}_0}{u_y} \tag{22}$$

Of course, this is not the true EV of the policy if the change in income results in changes in other equilibrium variables that affect the utility of the residents. For instance, consumers facing reduced incomes will alter their consumption of housing. In a partial equilibrium framework, this change in consumption would not cause the prices of housing and land to change. Our general equilibrium framework incorporates these potential price changes, changes which clearly affect consumer well-being. The reduced prices of housing as a result of reduced demand will offset consumers' loss of income, making the general equilibrium EV higher than the partial equilibrium effect noted in (22). The general equilibrium EV is calculated as

$$EV_{GE} = dy : \overline{u}(y,\ Restrictions) = \overline{u}(y + dy,\ No\ Restrictions). \tag{23}$$

Equation (23) can be solved numerically by solving the entire model again while treating $\overline{u}_0$ as an *exogenous* variable, and y as an *endogenous* variable. It can also be calculated using the comparative statics analysis, where $\frac{\partial \overline{u}}{\partial y}$ is calculated according to Appendix A.

$$dy = \frac{1}{\frac{\partial \overline{u}}{\partial y}} d\overline{u} \tag{24}$$

5 A Quantitative Application

The economic model described above, six equations in six unknowns (four unknown functions and two unknown parameters), can be solved using assumed functional forms for the production and consumer utility functions. The solution to the unconstrained model can be compared with the solution when lands are removed from the land supply for a portion of the area.

Assume that the utility function is Cobb-Douglas,

$$U(c, q) = c^{1-\alpha} \cdot q^{\alpha},\ 0 < \alpha < 1, \tag{25}$$

and the housing production function is also Cobb-Douglas,

$$h(S) = A \cdot S^{\gamma},\ 0 < \gamma < 1, A > 0. \tag{26}$$

Appendix B indicates how the model can be solved under these assumptions. Here we present the results from one application of the model. Our assumptions are noted below.

(1) Assume the utility function for consumers is Cobb-Douglas with $\alpha = 0.25$. Households devote one quarter of their incomes to housing expenditures, the income elasticity of demand for housing is one, and the elasticity of

substitution is also one. These stylized facts are consistent with survey and empirical evidence about consumer behavior, at least with respect to permanent income. See, for example, Goodman [1989], or Quigley [1979].

(2) Assume that the production function for housing is Cobb-Douglas with $\gamma = 0.70$, and $A = 1$. Thirty percent of the value of housing is accounted for by land and the remainder is capital improvements. This is roughly consistent with rules of thumb used in the assessment for property taxes (see, for example, Oates and Schwab [1997]) and with econometric evidence on production functions. See, for example, Muth [1971] or Quigley [1984]. This is also consistent with recent evidence on the elasticity of substitution in housing production (see Thorsnes [1997]).

(3) Assume that transportation costs for commuting are \$400 per mile per year. This represents a combination of out-of-pocket commuting costs and the cost of residents' time. The current mileage rate for business travel by private auto for tax purposes is \$0.36 per mile (see IRS Publication 463, 2004), while the remaining \$.44 per mile represents lost time due to due to commuting. At an average wage rate of \$30 an hour (see below) and a travel speed of 30-40 miles per hour, commuting time is assumed to be valued at about half of the wage rate.

(4) Assume that the income of households is \$60,000 per year.

(5) Assume that the rental value of land in the alternate use is \$250,000 per square mile, or just over \$400 per acre.

(6) Assume that the rental rate of capital, the real interest rate, is 3 percent.

(7) Assume that the region is expected to grow to a population of 400,000 households. Assuming 2.2 members per household means the region is about the size of the Tucson, AZ metropolitan area.

(8) We also assume that 33 percent of land area is used for residential housing, while the remaining two-thirds is used for alternative urban uses, streets, commercial areas, etc. This is consistent with estimates reported in widely used textbooks (e. g. Hartelson [1992]).

Under these stylized assumptions, we use the technique described in Appendix B to solve the model for the utility level of the residents, the geographic size of the developed region, and the spatial patterns of land rents, housing prices, housing consumption, and capital intensity. The solution to the model indicates that the built-up area extends for about 34.5 miles in each direction. The total built up area is about 3700 square miles, of which 33 percent is devoted to residential uses. The aggregate annual rent on the developed land is about \$1.6 billion or about \$2,000 per acre per year.

We now impose the requirement that about four percent of the land area be restricted, with residential construction forbidden, but the alternative use still permitted. At a distance of 32 miles from the center, we restrict $\pi/2$ radians in the region. Column 4 of Table 2 summarizes these effects. The geography of this designation is qualitatively identical to that in Figure 1, and Figures 3. A and 3. B.

Approximately 150 square miles have been restricted. This regulated land, which would otherwise have been developed for residential purposes, can now be used only in its alternative use, and merely earns the alternative rental rate of \$250,000 per square mile. The loss to the owners of these lands is just over \$2,000,000 a year, or about \$65 per acre of would-be housing in the regulated area. As a result of this regulation, the built-up region extends marginally further to about 34.8 miles in the rest of the region (i.e., in the $3\pi/2$ radians outside the designated area), creating roughly four square miles of new housing. The gain to the owners of these newly converted lands is about \$5,000 a year or about \$2 an acre on average. The owners of land that would have been developed regardless of the regulation gain because they develop their lands more intensively - since land is now scarcer. The annual change in rents on these lands is \$14.8 million or about \$19 per acre. The aggregate rents to all lands has increased by approximately \$12.7 million, or just under \$16 per acre. While overall landowner welfare has increased, consumers are made worse off. Consumer utility has decreased because land is scarcer, and they must pay more for housing while living in denser accommodations. Total losses to consumers, as measured by the equivalent variation of the restriction imposition, are just over \$15.5 million or about \$39 per household per year.

Table 2 also displays the economic effects of regulating other amounts of land, by varying the number of radians of restricted land but keeping the distance to the regulated land constant at 32 miles. As the number of radians of regulated land increases, the regulated area increases. The change in rents to lands which would otherwise have been developed regardless of the designation vary by as much as an order of magnitude, depending on the level of regulation.

Table 3 displays the impacts of various distances to the restricted lands, while assuming a constant "width" of $\pi/2$ radians. As reported in the table, when the boundary of the regulation is moved closer to the center of the region (holding the number of radians of restricted land constant) the restricted area increases. Increases in rents to land owners are large for those who occupy developable land. For those unfortunate enough to own regulated land, the losses can become quite substantial.

Table 2. The Economic Impacts of Varying Radians of Land Use Restrictions at a Distance of 32 Miles from the Center of the Region

	Radians of Land Use Restrictions						
	Baseline	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	π	$3\pi/2$
A. Geography							
Restricted Area (mi^2)	0.00	73.23	98.12	148.67	200.25	306.64	474.84
Built-Up Area (mi^2)	3,794.21	3,729.58	3,707.60	3,663.00	3,617.49	3,523.63	3,375.27
Percentage of Land Designated	0.00%	1.93%	2.58%	3.90%	5.25%	8.01%	12.33%
Miles to Urban-Rural Boundary	34.75	34.79	34.81	34.83	34.86	34.92	35.01
B. Change in Annual Rents to Land Owners							
Previously Developed Lands	$0	$7,295,025	$9,769,164	$14,783,460	$19,888,489	$30,384,504	$46,899,824
Newly Developed Lands	$0	$1,414	$2,420	$5,006	$8,086	$14,264	$17,186
Restricted Lands	$0	-$1,032,135	-$1,375,967	-$2,063,014	-$2,748,885	-$4,115,247	-$6,143,381
All Lands	$0	$6,264,304	$8,395,617	$12,725,452	$17,147,689	$26,283,520	$40,773,630
C. Change in Average Annual Land Rents Per Acre							
Previously Developed Lands	$0.00	$9.28	$12.51	$19.18	$26.14	$41.04	$66.06
Newly Developed Lands	$0.00	$0.89	$1.19	$1.81	$2.44	$3.75	$5.82
Restricted Lands	$0.00	-$66.74	-$66.40	-$65.70	-$65.00	-$63.54	-$61.26
All Lands	$0.00	$7.80	$10.45	$15.81	$21.27	$32.49	$50.14
D. Annual Equivalent Variation to Residents							
Aggregate	$0	-$7,674,396	-$10,283,654	-$15,581,273	-$20,987,160	-$32,137,845	-$49,768,378
Per Household	$0.00	-$19.19	-$25.71	-$38.95	-$52.47	-$80.34	-$124.42

Table 3. The Economic Impacts of Restricting Land Use for $\pi/2$ Radians of Land at Varying Distances from the Center of the Region

	Miles to Land Use Restrictions						
	Baseline	34.0	33.0	32.0	30.0	28.0	25.0
A. Geography							
Restricted Area (mi^2)	0.00	41.73	95.92	148.67	249.87	345.40	478.20
Built-Up Area (mi^2)	3,794.21	3,756.87	3,708.96	3,663.00	3,577.05	3,499.21	3,398.11
Percentage of Land Restricted	0.00%	1.10%	2.52%	3.90%	6.53%	8.98%	12.34%
Miles to Urban-Rural Boundary	34.75	34.77	34.80	34.83	34.90	34.98	35.13
B. Change in Annual Rents to Land Owners							
Previously Developed Lands	$0	$3,737,778	$9,052,662	$14,783,460	$27,575,410	$42,271,828	$68,183,620
Newly Developed Lands	$0	$317	$1,869	$5,006	$17,605	$41,894	$111,552
Restricted Lands	$0	-$149,240	-$822,827	-$2,063,014	-$6,350,188	-$13,224,394	-$28,831,659
All Lands	$0	$3,588,856	$8,231,704	$12,725,452	$21,242,827	$29,089,329	$39,463,514
C. Change in Average Annual Land Rents Per Acre							
Previously Developed Lands	$0.00	$4.71	$11.58	$19.18	$36.75	$57.82	$96.76
Newly Developed Lands	$0.00	$0.46	$1.11	$1.81	$3.40	$5.25	$8.58
Restricted Lands	$0.00	-$16.93	-$40.62	-$65.70	-$120.33	-$181.28	-$285.47
All Lands	$0.00	$4.47	$10.24	$15.81	$26.28	$35.83	$48.20
D. Annual Equivalent Variation to Residents							
Aggregate	$0	-$3,928,283	-$9,526,959	-$15,581,273	-$29,163,934	-$44,892,367	-$72,971,753
Per Household	$0.00	-$9.82	-$23.82	-$38.95	-$72.91	-$112.23	-$182.43

5.1 Partial vs. General Equilibrium Impacts of Land Use Restrictions

This section describes how our model of the general equilibrium effects of land use restrictions compares to a partial equilibrium assessment. A partial equilibrium approach assumes that no other prices change in response to the designation. This is formally equivalent to the “open region” assumption discussed in the introduction. In such a world, the only welfare effects of regulation are the lost land rents to the owners of designated lands. The residents that would otherwise live in restricted lands instead move outside of the region.

In our “closed region” model, we assume that the region’s residents do not move out of the system. These displaced residents then change the demand for land and housing in the remainder of the region, causing price changes throughout the system. These price changes lead to other welfare losses in addition to the lost rents which would otherwise be earned on the designated lands. Our methodology recognizes the impacts of land use restrictions on previously developed lands, newly developed lands, and consumers (see Table 4).

As Table 4 shows, the partial equilibrium approach estimates the total impacts of the restriction to be -$2,063,014, while the general equilibrium approach estimates the total impacts to be -$2,855,821, a difference of 38 percent. The most important limitation of a partial equilibrium approach, however, is that it ignores the large transfers that may result from land use restrictions. The general equilibrium approach shows that the total impacts are greater than under the partial equilibrium approach, but, more importantly, it also shows that there are nearly $15 million dollars transferred from consumers to non-regulated landowners in the region. This underscores an important part of the analysis of the impacts of land use restrictions: the net impacts are small in

Table 4: Partial vs. General Equilibrium Impacts of Land Use Restriction: Regulating $\pi/2$ radians of land at 32 miles from the region's center.

	Approach		
Including Impacts on…	Partial	General	Economic Impact
Restricted Lands	√	√	-$2,063,014
Previously Developed Lands		√	$14,783,460
Newly Developed Lands		√	$5,006
Consumers		√	-$15,581,273
Total	-$2,063,014	-$2,855,821	

comparison to the wealth transfers created by the policy. [5]

6 Extension: Urban Growth Boundary

This section extends the model to the case in which the regulation completely surrounds the region and forces the displaced residents to live within the previously developed and unregulated lands. In this case, the restriction becomes a growth boundary. In Section 4 we modelled the land use restriction impacts as a function of the distance to the designation, x^* and the radians of regulated land, k. In this section we examine the impacts of a policy that sets $k = 2\pi$, in other words, a policy that prevents all development beyond an urban growth boundary located at x^*.

Whereas previous analysis allowed for the development of previously undeveloped lands, extending the urban-rural boundary, a growth boundary regulation forces the displaced residents to live on the previously developed but unregulated lands. Under a growth boundary policy, the urban-rural boundary is no longer determined endogenously. In fact, the boundary of the built up region is determined by $\overline{x} = x^*$. With an exogenous boundary condition,

[5] Of course, if the residents owned the land in common, the wealth transfers reported in Table 4 would be largely eliminated. More realistically, about a third of metropolitan residents are renters, and they are made substantially worse off by the policy.

the system is simply determined by the five equations (27)–(31).

$$\frac{U_2(y - p(x)q(x) - tx, q(x))}{U_1(y - p(x)q(x) - tx, q(x))} = p(x) \tag{27}$$

$$max\ U(y - p(x)q(x) - tx, q(x)) = \overline{u}. \tag{28}$$

$$p(x)h'(S(x)) = i \tag{29}$$

$$p(x)h(S(x)) - iS(x) = r(x) \tag{30}$$

$$\int_0^{x^*} 2\pi x \frac{h(S(x))}{q(x)} dx = N \tag{31}$$

Table 5 displays the impacts of growth boundaries at differing distances from the center of the region rather than the initial equilibrium 34. 5 miles to the urban-rural border. The table shows results similar to Tables 2 and 3, the losses to regulated lands are the most intense, but there are much larger effects upon consumers and the owners of unregulated land.

Table 5. The Economic Impacts of Growth Boundaries at Varying Distances from the Center of the Region

	Distance to Growth Boundary (in miles)				
	34	33	32	30	25
% of Developed Area Regulated	4.3%	9.8%	15.2%	25.5%	48.3%
Change in Annual Rents to Land Owners					
Inside Growth Boundary	$16,234,093	$39,387,454	$64,442,270	$120,695,100	$302,164,590
Outside Growth Boundary	-$597,382	-$3,293,790	-$8,258,693	-$25,423,977	-$115,471,490
All Lands	$15,636,711	$36,093,664	$56,183,578	$95,271,127	$186,693,100
Change in Average Annual Land Rents Per Acre					
Inside Growth Boundary	$21.17	$54.51	$94.85	$202.12	$728.65
Outside Growth Boundary	-$17.40	-$41.81	-$67.75	-$124.52	-$298.65
All Lands	$20.39	$49.95	$82.69	$159.54	$450.20
Annual Equivalent Variation to Residents					
Aggregate	-$17,102,303	-$41,695,390	-$68,585,851	-$130,076,900	-$340,561,470
Per Household	-$42.76	-$104.24	-$171.46	-$325.19	-$851.40

Brueckner [2001] contains a simplified model of land consumption in an examination of growth boundaries. His analysis of policy solutions to combat urban sprawl highlights the difficulties of using these regulations to correct market failures from urban sprawl. He notes that overzealous enaction of growth boundaries may create larger costs than benefits and recommends the use of development fees and congestion taxes to internalize the externalities of sprawl. Our model extends his work to housing but does not consider externalities from urban development. Nevertheless, it suggests that the costs from exogenously imposed growth boundaries can be quite high indeed.

7 Conclusion

This paper analyzes the economic consequences of land use regulation, thus restricting its economic uses. When the amount of regulated land is significant, and the land would otherwise have been used to produce housing, the regulations will have effects upon the equilibrium of the local economy, its land and housing markets. The reduction in the land available for development means that other land, which would not have been developed for housing, can now be profitably developed. Still other land, which would have been developed at lower densities, is instead developed more intensely. These lands increase in value, and the rents to land owners increase, offsetting in part, the reduced value and rent of regulated lands. The well being of consumers declines as housing prices and densities increase.

We present a flexible general equilibrium model of these interactions. The model is highly stylized: the built-up area is initially circular; and the land use restriction is modelled as some portion of an annulus of a given width at a given distance from the center of the region. In this way, the economic effects of increasing the quantity of land regulation, or of restricting more valuable land (closer to the center), can be investigated. We assume that the region is a closed economy, meaning that the population is exogenous and fixed, and we compare static equilibria with and without the regulation. The model focuses upon the impacts upon the region from changes in the supply of land and their resulting impacts throughout the economy. The model does not include other benefits that may accrue to the region, such as increased utility from the preservation of open space.

Although the model is highly stylized, it can be adapted to a variety of circumstances, and it can be used to compare the effects of land use restrictions in regions of differing sizes and with varying levels and densities of economic activity. The model is calibrated using plausible functional forms and initial conditions, and the model is exercised in a single example. The simulation results illustrate the importance of the indirect effects in assessing the costs of land use restrictions. If more than a small percentage of the region's land is regulated, then the most important economic consequences of the regulations are not their effects upon restricted lands. Rather, the most important consequences are the increases in the rents and prices of land which would have been developed anyway. This leads to losses to consumers who must face higher housing prices.

The simulation results suggest that land use restrictions can cause large and significant redistribution of welfare among land owners and consumers in a metropolitan region. When regulated lands are located close to the periphery of the region, the loss to the owners of the restricted lands may be three or four times larger, per acre of land than the gains to the owners of land which would have been developed anyway. However, even when the restricted area is only a few percent of the land area of the region, the aggregate gain to owners of land which would have been developed anyway — in the absence of the land use restrictions — is much larger than the aggregate loss to the owners of restricted lands. These results mirror the impacts to consumers: although the economic effects upon each consumer are small, in aggregate they may overshadow the economic effects upon regulated lands.

As the restricted land is moved closer to the center, the land designated has a higher opportunity cost, since it would have been used more intensely for housing production. Thus, the losses to the owners of these lands are much larger. But, in the simulations explored in this paper, in the aggregate these losses are still a good bit less than the aggregate gains to the owners of other land which would have been developed anyway.

The principal distributional effect of these regulations is to reduce the well being of housing consumers in the region. When small amounts of land are reguylated, and when these lands are located near the periphery, it is nevertheless true that the aggregate losses to consumers are about ten times as large as the aggregate gains to landowners. This relative relationship is maintained when the land designated is located closer to the center. (But, of course, the aggregate losses when more valuable land is regulated are much larger.)

This paper shows the importance of examining the land use restrictions in a general equilibrium framework; the most important impacts may occur to non-restricted lands, and the region's residents. The numerical results vary with the particular scenarios which are simulated. However, even when reasonably small areas of the region are restricted, and even when these areas are located close to the periphery, the numerical results predict that the losses to consumers in the region are not negligible. This underscores the basic fact that policy makers wishing to compare the economic costs of land use restrictions with its benefits must not only examine the impacts upon the regulated landowners, but also the region's residents, who are affected through higher prices of housing and other landowners who benefit from increased demand for their land.

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Appendices

A Comparative Statics Derivation

Land use restrictions alter the spatial development of a region, changing the system of equations governing the spatial equilibrium. We can analyze the impacts of changes in the system's exogenous variables, including land use restrictions, using comparative statics analysis. It is convenient to use Cramer's Rule; to do this we totally differentiate the six equations in the system, equations (3), (4), (7), (8), (9) and (12). The system of total differentials in matrix notation is:

$$\begin{bmatrix} -qv_{21} - v_1 + pqv_{11} & -2pv_{21} + v_{22} + p^2 v_{11} & 0 & 0 & 0 & 0 \\ -qv_1 & -pv_1 + v_2 & 0 & 0 & -1 & 0 \\ h'(S(x)) & 0 & ph''(S(x)) & 0 & 0 & 0 \\ h(S(x)) & 0 & ph'(S(x)) - i & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & r'(\overline{x}) \\ 0 & \begin{pmatrix} -\int_0^{x^*} \frac{2\pi x h(S(x))}{q^2(x)} dx - \\ \int_{x^*}^{\overline{x}} \frac{(2\pi - k) x h(S(x))}{q^2(x)} dx \end{pmatrix} & \begin{pmatrix} \int_0^{x^*} \frac{2\pi x h'(S(x))}{q(x)} dx + \\ \int_{x^*}^{\overline{x}} \frac{(2\pi - k) x h'(S(x))}{q(x)} dx \end{pmatrix} & 0 & 0 & \frac{(2\pi - k)\overline{x} h(S(\overline{x}))}{q(\overline{x})} \end{bmatrix} \begin{bmatrix} dp \\ dq \\ dS \\ dr \\ d\overline{u} \\ d\overline{x} \end{bmatrix}$$

$$= \begin{bmatrix} -v_{21} + pv_{11} & xv_{21} - pxv_{11} & 0 & 0 & 0 & 0 & 0 \\ -v_1 & xv_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & S & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \frac{kxh(S(x^*))}{q(x^*)} & -\int_{x^*}^{\overline{x}} \frac{xh(S(x))}{q(x)} dx \end{bmatrix} \begin{bmatrix} dy \\ dt \\ di \\ dN \\ dx^* \\ dk \end{bmatrix} \tag{A.1}$$

Standard assumptions regarding the shape of the consumer utility and housing production functions are sufficient to determine the algebraic sign of the components of these matrices. Assume the standard conditions of positive, but diminishing, marginal utility from consumption, and gross substitution

$$v_1 > 0,\ v_2 > 0,\ v_{11} < 0,\ v_{22} < 0,\ and\ v_{12} = v_{21} > 0, \tag{A.2}$$

and analogous conditions of diminishing marginal productivity

$$h'(S(x)) > 0,\ h''(S(x)) < 0. \tag{A.3}$$

This allows us to sign each element in the matrices of equation (A.1).

$$\begin{bmatrix} - & - & 0 & 0 & 0 & 0 \\ - & 0 & 0 & 0 & - & 0 \\ + & 0 & - & 0 & 0 & 0 \\ + & 0 & 0 & - & 0 & 0 \\ 0 & - & + & 0 & 0 & - \\ 0 & - & + & 0 & 0 & + \end{bmatrix} \begin{bmatrix} dp \\ dq \\ dS \\ dr \\ d\overline{u} \\ d\overline{x} \end{bmatrix} = \begin{bmatrix} - & + & 0 & 0 & 0 & 0 & 0 \\ - & + & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & + & 0 & 0 & 0 & 0 \\ 0 & 0 & + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & + & 0 & 0 \\ 0 & 0 & 0 & + & 0 & - & + \end{bmatrix} \begin{bmatrix} dy \\ dt \\ di \\ dN \\ dr_a \\ dx^* \\ dk \end{bmatrix} \tag{A.4}$$

These matrices are sufficient to determine the direction of changes in the endogenous variables arising from changes in the exogenous parameters. What follows is a description of the comparative statics results for the economic model used in the analysis.

First, what are the impacts of increasing consumer income? Equation (A.5) shows that an increase in consumer's income will increase demand for housing, the price of housing, housing densities, land prices, resident utility level, and will cause the city to expand.

$$\frac{\partial p}{\partial y} > 0,\ \frac{\partial q}{\partial y} > 0,\ \frac{\partial S}{\partial y} > 0,\ \frac{\partial r}{\partial y} > 0,\ \frac{\partial \overline{u}}{\partial y} > 0,\ \frac{\partial \overline{x}}{\partial y} > 0. \tag{A.5}$$

Since commuting costs are simply income reductions in this formulation, it is clear that increases in travel costs are exactly equivalent to decreases in income. As such, all comparative statics results in (A.6) are of the opposite sign of those in (A.5).

$$\frac{\partial p}{\partial t} < 0,\ \frac{\partial q}{\partial t} < 0,\ \frac{\partial S}{\partial t} < 0,\ \frac{\partial r}{\partial t} < 0,\ \frac{\partial \overline{u}}{\partial t} < 0,\ \frac{\partial \overline{x}}{\partial t} < 0. \tag{A.6}$$

Equation (A.7) describes the effects of increases in the price of capital. We obtain the intuitive results that when input costs increase, housing prices increase and quantity demanded decreases, and consumer utility decreases. However, we cannot determine the impacts upon housing density, land prices, or the size of the city, without making further assumptions regarding the production function.

$$\frac{\partial p}{\partial i} > 0,\ \frac{\partial q}{\partial i} < 0,\ \frac{\partial S}{\partial i} =?,\ \frac{\partial r}{\partial i} =?,\ \frac{\partial \overline{u}}{\partial i} < 0,\ \frac{\partial \overline{x}}{\partial i} =?. \tag{A.7}$$

Equation (A.8) indicates that an increase in the population of the closed region will cause housing prices to rise, the quantity of housing demanded by each consumer to fall, which reduces consumer utility. Housing densities will increase, causing land prices to increase, which causes the size of the city to increase.

$$\frac{\partial p}{\partial N} > 0,\ \frac{\partial q}{\partial N} < 0,\ \frac{\partial S}{\partial N} > 0,\ \frac{\partial r}{\partial N} > 0,\ \frac{\partial \overline{u}}{\partial N} < 0,\ \frac{\partial \overline{x}}{\partial N} > 0. \tag{A.8}$$

At this point we can examine the effects of changes in the regulation parameters. Recall that x^* is a measure of the distance from the restricted land to the center of the region. An increase in x^* represents "moving" the restricted land area to a location farther from the city. As intuition would predict, this reduces the impacts from the designation process. Equation (A.9) shows that increasing x^* decreases the price of housing, increases the demand for housing, decreases residential density, decreases land rents, increases consumer welfare, and reduces the distance to the urban-rural boundary.

$$\frac{\partial p}{\partial x^*} < 0,\ \frac{\partial q}{\partial x^*} > 0,\ \frac{\partial S}{\partial x^*} < 0,\ \frac{\partial r}{\partial x^*} < 0,\ \frac{\partial \overline{u}}{\partial x^*} > 0,\ \frac{\partial \overline{x}}{\partial x^*} < 0. \tag{A.9}$$

The other land use restriction parameter, k, represents the number of radians of land that have been removed from the supply of land for urban uses. Increasing k increases the amount of restricted land (and, at the extreme, $k = 2\pi$, fully surrounds the city). Equation (A.10) indicates that an increase in k will cause housing prices to rise, and the quantity of housing demanded by each consumer to fall, both of which combine to reduce consumer utility. Housing densities will increase, causing land prices to increase, which causes the distance to the rural-urban boundary to increase.

$$\frac{\partial p}{\partial k} > 0,\ \frac{\partial q}{\partial k} < 0,\ \frac{\partial S}{\partial k} > 0,\ \frac{\partial r}{\partial k} > 0,\ \frac{\partial \overline{u}}{\partial k} < 0,\ \frac{\partial \overline{x}}{\partial k} > 0. \tag{A.10}$$

B Solving the Model with Cobb-Douglas Utility and Production Functions

In this section we numerically assess the impact of land use restrictions by assuming simple Cobb-Douglas functional forms for consumer utility and housing production. We first calculate the initial equilibrium conditions in a region an unregulated economy and then calculate the prevailing equilibrium conditions for an otherwise identical but regulated region.

We assume the following Cobb-Douglas functional forms:

$$U(c, q) = c^{1-\alpha} \cdot q^{\alpha},\ 0 < \alpha < 1 \tag{B.1}$$

$$h(S) = A \cdot S^{\gamma},\ 0 < \gamma < 1 \tag{B.2}$$

With this formulation, households spend fraction α of their incomes on housing services, the income elasticity of housing is one, and land expenditures represent fraction γ of the total housing production costs. The first step is to solve the six equations governing the unregulated economy simultaneously—equations (3), (4), (7), (8), (9), and (10). Substituting equations (B.1) and (B.2) into (3), (4), (7), (8), (9), and (10) yields the six equilibrium conditions below.

$$\frac{U_2(y - p(x)q(x) - tx, q(x))}{U_1(y - p(x)q(x) - tx, q(x))} = p(x) = \frac{\alpha c(x)^{1-\alpha} q(x)^{\alpha-1}}{(1-\alpha)c(x)^{1-\alpha-1} q(x)^{\alpha}} \tag{B.3}$$

$$U\left(c(x), q(x)\right) = \overline{u} = c(x)^{1-\alpha} * q(x)^{\alpha} \tag{B.4}$$

$$p(x)h'(S(x)) = i = p(x)A\gamma \left[S(x)\right]^{\gamma-1} \tag{B.5}$$

$$p(x)h(S(x)) - iS(x) = r(x) = p(x)A\left[S(x)\right]^{\gamma} - iS(x) \tag{B.6}$$

$$r(\overline{x}) = r_a \tag{B.7}$$

$$\int_0^{\overline{x}} 2\pi x \frac{h(S(x))}{q(x)} dx = N = \int_0^{\overline{x}} 2\pi x A\left[S(x)\right]^{\gamma} q(x)^{-1} dx \tag{B.8}$$

We can use Walras' Law and successive substitution to render equations (B.3)–(B.6) as a functions of $\overline{u}$ and exogenous parameters. Then, we can use (B.6) to solve (B.7) for $\overline{u}$ as a function of $\overline{x}$, which, when substituted into (B.8), yields one equation and one unknown, $\overline{x}$. After solving this single equation for the equilibrium size of the built up region, we can calculate the equilibrium citizen utility level, and solve for the four functions that fully describe the equilibrium.

Walras' Law states that total expenditures must equal total income

$$c(x) + p(x)q(x) = y - tx \Rightarrow c(x) = y - tx - p(x)q(x). \tag{B.9}$$

When substituted into (B.3), and rearranged, this yields

$$q(x) = \alpha(y - tx)\left[p(x)\right]^{-1}. \tag{B.10}$$

Now substitute this result into (B.4) to solve for $p(x)$ as a function of $\overline{u}$.

$$p(x) = \alpha(1-\alpha)^{(\frac{\alpha-1}{\alpha})}(y-tx)^{(\frac{1}{\alpha})}\overline{u}^{(\frac{-1}{\alpha})}. \tag{B.11}$$

When combined with (B.10), this yields $q(x)$ as a function of $\overline{u}$.

$$q(x) = (1-\alpha)^{(\frac{\alpha-1}{\alpha})}(y-tx)^{(\frac{\alpha-1}{\alpha})}\overline{u}^{(\frac{1}{\alpha})}. \tag{B.12}$$

Together, (B.11) and (B.12) describe the equilibrium consumer behavior. Substituting (B.11) into equation (B.5) yields $S(x)$ as a function of $\overline{u}$.

$$[p(x)]\,\gamma A\,[S(x)]^{\gamma-1} = i, \tag{B.13}$$

$$\left[\alpha(1-\alpha)^{(\frac{\alpha-1}{\alpha})}(y-tx)^{(\frac{1}{\alpha})}\overline{u}^{(\frac{-1}{\alpha})}\right]\gamma A\,[S(x)]^{\gamma-1} = i, \tag{B.14}$$

$$S(x) = \left[\alpha(1-\alpha)^{(\frac{1-\alpha}{\alpha})}(y-tx)^{(\frac{1}{\alpha})}\overline{u}^{(\frac{-1}{\alpha})}A\gamma i^{-1}\right]^{\frac{1}{1-\gamma}}, \tag{B.15}$$

Substitute this result into (B.6) to get $r(x)$ as a function of $\overline{u}$.

$$[p(x)]\,A\,[S(x)]^{\gamma} - i\,[S(x)] = r(x), \tag{B.16}$$

$$\begin{aligned}&\left[\alpha(1-\alpha)^{(\frac{\alpha-1}{\alpha})}(y-tx)^{(\frac{1}{\alpha})}\overline{u}^{(\frac{-1}{\alpha})}\right]A\left[\left(\alpha(1-\alpha)^{(\frac{1-\alpha}{\alpha})}(y-tx)^{(\frac{1}{\alpha})}\overline{u}^{(\frac{-1}{\alpha})}A\gamma i^{-1}\right)^{\frac{1}{1-\gamma}}\right]^{\gamma}\\ &-i\left[\alpha(1-\alpha)^{(\frac{1-\alpha}{\alpha})}(y-tx)^{(\frac{1}{\alpha})}\overline{u}^{(\frac{-1}{\alpha})}A\gamma i^{-1}\right] = r(x).\end{aligned} \tag{B.17}$$

This expression simplifies to

$$r(x) = \left[\alpha(1-\alpha)^{(\frac{1-\alpha}{\alpha})}(y-tx)^{(\frac{1}{\alpha})}\overline{u}^{(\frac{-1}{\alpha})}Ai^{-\gamma}\gamma\right]^{\frac{1}{1-\gamma}}(\gamma^{-1}-1). \tag{B.18}$$

At this point we have two remaining equations, (B.7 and B.8), and two unknowns, $\overline{x}$ and $\overline{u}$.

Substituting (B.18) into

$$r(\overline{x}) = r_a, \tag{B.19}$$

yields

$$\left[\alpha(1-\alpha)^{(\frac{1-\alpha}{\alpha})}(y-t\overline{x})^{(\frac{1}{\alpha})}\overline{u}^{(\frac{-1}{\alpha})}Ai^{-\gamma}\gamma\right]^{\frac{1}{1-\gamma}}(\gamma^{-1}-1)=r_a, \tag{B.20}$$

solving for $\overline{u}$,

$$\overline{u}=r_a^{-\alpha(1-\gamma)}\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}(y-t\overline{x})A^{\alpha}i^{(-\alpha\gamma)}\gamma^{\alpha}(\gamma^{-1}-1)^{\alpha(1-\gamma)}. \tag{B.21}$$

Meanwhile, substituting (B.15) and (B.12) into

$$N=\int_0^{\overline{x}}2\pi xA\left[S(x)\right]^{\gamma}\left[q(x)\right]^{-1}dx, \tag{B.22}$$

yields

$$N=2\pi A\left[(1-\alpha)^{(\frac{1-\alpha}{\alpha})}\left(\frac{\alpha\gamma}{i}\right)^{\gamma}\right]^{\frac{1}{1-\gamma}}\overline{u}^{(\frac{-1}{\alpha-\alpha\gamma})}\int_0^{\overline{x}}x(y-tx)^{(\frac{1-\alpha+\alpha\gamma}{\alpha-\alpha\gamma})}dx. \tag{B.23}$$

Replacing equation $\overline{u}$ in (B.23) with the RHS of (B.21) yields equation (B.24), with only one unknown, $\overline{x}$.

$$N=2\pi r_a\alpha^{(\frac{\alpha-1}{1-\gamma})}\gamma^{-1}(\gamma^{-1}-1)^{-1}(y-t\overline{x})^{(\frac{-1}{\alpha-\alpha\gamma})}\int_0^{\overline{x}}x(y-tx)^{(\frac{1-\alpha+\alpha\gamma}{\alpha-\alpha\gamma})}dx. \tag{B.24}$$

Equation (B.24), one equation and one unknown, can be solved for the equilibrium $\overline{x}$ using integration by parts. This solution can then be used in equation (B.21) to find the equilibrium utility level, and from this, the remaining functions p(x), q(x), S(x), and r(x) can be solved explicitly.

B.1 Land Use Restrictions

Restricting the use of some lands replaces equation (B.8) with

$$\int_0^{x^*}2\pi x\frac{h(S(x))}{q(x)}dx+\int_{x^*}^{\overline{x}}(2\pi-k)x\frac{h(S(x))}{q(x)}dx=N \tag{B.25}$$

or

$$\int_0^{x^*} 2\pi x A \left[S(x)\right]^\gamma q(x)^{-1} dx + \int_{x^*}^{\overline{x}} (2\pi - k) x A \left[S(x)\right]^\gamma q(x)^{-1} dx = N \quad \text{(B.26)}$$

Substituting equations (B.12), (B.15), and (B.21) into (B.26) again yields one equation and one unknown. This equation can be solved for the new equilibrium $\overline{x}$, the resulting $\overline{u}$, and functions $p(x),\ q(x),\ S(x),\ and\, r(x)$. Using these results we can more fully understand the impacts of land use restrictions.