Optimal Index versus Simple Index for Monetary Policy

Jae Won Lee   Yeji Sung

Very preliminary

Introduction 1

- Classic questions in monetary policy literature
- Desirable to stabilize inflation?
  - Inefficient price dispersion at micro level
- What index to use?
  - In a standard one-sector model, use simple standard index
  - e.g. CPI
- With multiple sector with heterogeneities?
  - Price rigidities
  - Labor shares
  - Home vs. Foreign
  - etc.

Introduction 2

- Influential results from Aoki (2001), Benigno (2004) and others
  - *Simple* inflation targeting (SIT) may be far from optimal.
- Different products
  - Different degree of competitiveness (and price stickiness)
- "Stickiness principle"
  - Disproportionately larger weight on sticky goods
  - Target *optimal* index rather than *simple* index
- *Optimal* inflation targeting (OIT) is
  - not only better than SIT
  - but also quite close to the optimal policy (hard to implement in real world)

Introduction 3

- In other words,

- Don't stabilize CPI:

$$\pi = n_1\pi_1 + n_2\pi_2$$

- Instead, construct and stabilize

$$\tilde{\pi} = \delta_1\pi_1 + \delta_2\pi_2$$

with

$$\begin{aligned} \delta_1 &\gg n_1 \\ \delta_2 &\ll n_2 \end{aligned}$$

Introduction 4

- A great and simple idea!
  - But, implementation is not so simple
  - In reality, monetary policy is closer to SIT.
- How can we reconcile such the disconnection?
- We explore two potential reasons.
  1. $OIT \succ SIT$, but welfare gain may be small.
  2. OIT may be politically infeasible.

Political feasibility of OIT

- Moving $\delta \rightarrow$ distributional implications
- Different goods are often produced by different people in different regions (comparative advantage)
- German and France specialize in different goods (Car vs. Wine)
- Suppose French goods' prices are overall stickier (not true in reality)
- *OIT* would put much larger weight on π_{france} than $\pi_{germancy}$
- Perhaps, infeasible politically.

Welfare gain

- By definition, $OIT \succ SIT$
- But under what conditions,
  - is SIT way below OIT?
  - is SIT is relatively fine?
- Focus on frictions between member countries in CU
  - Financial and real market integration may still not be ideal.
  - The magnitude of benefit of switching to OIT from SIT depends on the degree of union integration.
  - Financial integration
    - Frictions in borrowing/lending $\Longrightarrow$ SIT is okay!
  - Real integration
    - Frictions in importing/exporting $\Longrightarrow$ SIT is bad!

Model outline

- Two countries $\{A, B\}$ in CU $[0, 1]$
  - $\mathcal{I}_a = [0, n_a]$ and $\mathcal{I}_B = (n_a, 1]$.
- Produce different goods
  - trade their products and state-contingent assets
- Cross-country flows of goods and funds however are subject to certain frictions
- Characterized by different degree of price stickiness

- Single central bank
- Fiscal policy is not sophisticated
  - Collect lump-sum taxes and use them to rectify (steady-state) distortions from frictions
  - but do not employ state-contingent policy

Model

- Focus on country A representative household

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_{a,t}) - \frac{1}{n_a} \int_{\mathcal{I}_A} V(N_{a,t}(i))\, di \right]$$

$$\text{s.t. } P^C_{a,t} C_{a,t} + P^C_{a,t} \mathbb{E}_t[Q_{t,t+1} B_{a,t+1}] + P^C_{a,t} \Phi(C_{a,t}, \xi_{a,t}) = P^C_{a,t} B_{a,t} + P^C_{a,t} \xi_{a,t}$$

$$\begin{aligned} P^C_{a,t} \xi_{a,t} \quad &\equiv \quad \frac{1}{n_a} \int_{\mathcal{I}_a} W_{a,t}(i) N_{a,t}(i) di + W^T N^T_{a,t} \\ &\quad + \frac{1}{n_a} \int_{\mathcal{I}_a} \Pi_{a,t}(i) di + \Pi^F_{a,t} + \Pi^T_{a,t} - P^C_{a,t} \tau_{a,t} \end{aligned}$$

- Financial frictions

$$\Phi(C_{a,t}, \xi_{a,t}) = \phi \frac{C_{a,t}}{2} \left(\log \frac{C_{a,t}}{\xi_{a,t}} \right)^2 \quad \text{where} \quad \phi \geq 0$$

Consumption basket and CPI

- Consumption basket

$$C_{a,t} = \left[{n_a}^{\frac{1}{\eta}} {C_{a,a,t}}^{\frac{\eta-1}{\eta}} + {n_b}^{\frac{1}{\eta}} {C_{a,b,t}}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

- CPI

$$P^C_{a,t} = \left[n_a {P_{a,t}}^{1-\eta} + n_b \tilde{P}_{b,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

- Trade frictions
  - LOP does not hold

$$\tilde{P}_{b,t} \neq P_{b,t}; \qquad P_{a,t} \neq \tilde{P}_{a,t}$$

  - PPP does not hold

$$P^C_{a,t} \neq P^C_{b,t}$$

Trade frictions

- Importing agency imports $M_{a,t} = f(N^T_{a,t})$
  - requires hiring $N^T_{a,t}$ of labor services
- How much to import?

$$\begin{aligned} \max_{M_{a,t}} \Pi^T_{a,t} &= \left(\tilde{P}_{b,t} - P_{b,t} + s^T\right) M_{a,t} - W^T N^T_{a,t} \\ \text{s.t. } M_{a,t} &= f(N^T_{a,t}) \end{aligned}$$

- Market clearing $M_{a,t} = C_{a,b,t}$.
- Import price is higher when country-A's household demand more of country-B's products.

$$\tilde{P}_{b,t} = P_{b,t} + \underbrace{\omega\left(C_{a,b,t}\right)}_{\text{endogenous wedge}}$$

Price rigidities

- $P_{a,t}$ and $P_{b,t}$ are sticky in a standard way (Calvo)

$$P^*_{a,t} = \arg\max_{P_{a,t}(i)} \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha_a \beta)^k \, Q_{t,t+k} \underbrace{\begin{bmatrix} P_{a,t}(i) Y_{a,t+k}(i) \\ -(1-s) W_{a,t+k}(i) N_{a,t+k}(i) \end{bmatrix}}_{\Pi_{a,t+k}(i)}$$

- Production function

$$Y_{a,t}(i) = Z_{a,t} N_{a,t}(i)$$

  - $Z_{a,t}$ is country A productivity shock

$$\log Z_{j,t} = \rho_j \log Z_{j,t-1} + \varepsilon_{j,t}$$

- Country A's PPI

$$P_{a,t} = \left[(1-\alpha_a) \left(P^*_{a,t} \right)^{1-\theta} + \alpha_a \left(P_{a,t-1} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

Market clearing conditions

- Asset markets

$$n_a B_{a,t} + n_b B_{b,t} = 0$$

- Import supply and demand

$$\begin{aligned} M_{a,t} &= C_{a,b,t} \\ M_{b,t} &= C_{b,a,t} \end{aligned}$$

- Goods market

$$\begin{aligned} Y_{a,t} &= n_a C_{a,a,t} + n_b C_{b,a,t} \\ Y_{b,t} &= n_a C_{a,b,t} + n_b C_{b,b,t} \end{aligned}$$

Additional notations

- Union-wide & relative variables

$$\begin{aligned} X_t^U &= n_a X_{a,t} + n_b X_{b,t} \\ X_t^R &= \frac{X_{a,t}}{X_{b,t}} \end{aligned}$$

- Efficient allocation

$$X_t^E$$

- LQ approach

$$\hat{X}_t = \log X_t - \log \bar{X}$$

$$\begin{aligned} \hat{X}_t^U &= n_a \hat{X}_{a,t} + n_b \hat{X}_{b,t} \\ \hat{X}_t^R &= \hat{X}_{a,t} - \hat{X}_{b,t} \end{aligned}$$

Efficient allocation

- Utilitarian social planner
  - or market outcome *without* 'nominal,' 'financial,' 'trade' frictions

$$
\begin{aligned}
Y^{U,E} &= C^{U,E} \equiv n_a C_a^E + n_b C_b^E = \left[\sum_{a,b} n_j Z_j^{\frac{(\eta-1)(1+\varphi)}{1+\varphi\eta}} \right]^{\frac{1+\varphi\eta}{(\eta-1)(\sigma+\varphi)}} \\
Y^{R,E} &\equiv \frac{Y_a^E}{Y_b^E} = \frac{n_a}{n_b} \left(\frac{Z_a}{Z_b} \right)^{\frac{\eta(1+\varphi)}{1+\varphi\eta}} \\
C^{R,E} &\equiv \frac{C_a^E}{C_b^E} = 1 \\
\frac{C_{a,a}^E}{C_{a,b}^E} &= \frac{C_{b,a}^E}{C_{b,b}^E} = Y^{R,E}
\end{aligned}
$$

  - Relation between relative price and output

$$
Y^{R,E} = \frac{n_a}{n_b} \left(P^{R,E} \right)^{-\eta}
$$

Market allocation (in first order approximation)

- Financial frictions: $\phi \geq 0$
- Relative consumption (imperfect risk-sharing)

$$\hat{C}_t^R = \frac{\phi}{\sigma + \phi}\hat{\xi}_t^R$$

- If $\phi = 0$

$$\hat{C}_t^R = 0 \;\; (= \hat{C}^{R,E})$$

- Trade frictions ($\nu \geq 0$)

$$\tilde{P}_{b,t} = \hat{P}_{b,t} + \nu \hat{C}_{a,b,t}; \qquad \tilde{P}_{a,t} = \hat{P}_{a,t} + \nu \hat{C}_{b,a,t}$$

$$\hat{P}^R_{a,t} \equiv \hat{P}_{a,t} - \hat{\tilde{P}}_{b,t}; \qquad \hat{P}^R_{b,t} \equiv \hat{\tilde{P}}_{a,t} - \hat{P}_{b,t}$$

  - If no frictions ($\nu = 0$)

$$\hat{P}^R_{a,t} = \hat{P}^R_{b,t} = \hat{P}^R_t \left(\equiv \hat{P}_{a,t} - \hat{P}_{b,t} \right)$$

- $\hat{P}^R_{a,t} \neq \hat{P}^R_{b,t}$ generates distortions in
  - (i) Cross-country relative output

$$\hat{Y}^R_t = -\eta \left(n_a \hat{P}^R_{a,t} + n_b \hat{P}^R_{b,t} \right) = -\eta \Lambda_D \hat{P}^R_t \qquad \Lambda_D \in (0,1]$$

    - $\Lambda_D = 1$ with no trade frictions
    - $\Lambda_D < 1$ with trade frictions (**muted** response of relative output)
  - (ii) within-country consumption bundle

$$\widehat{\frac{C_{a,a,t}}{C_{a,b,t}}} - \widehat{\frac{C_{b,a,t}}{C_{b,b,t}}} = -\eta \left(\hat{P}^R_{a,t} - \hat{P}^R_{b,t} \right)$$

Nominal frictions

- Country PPI evolves as:

$$\hat{P}_{a,t} = (1-\alpha_a)\hat{P}^{*}_{a,t} + \alpha_a \hat{P}_{a,t-1}$$

- Phillips curves

$$\pi_{a,t} = \beta \mathbb{E}_t \pi_{a,t+1} + \kappa_a \widehat{mc}_{a,t}$$

$$\widehat{mc}_{a,t} = (\sigma+\varphi)\hat{Y}^U_t + n_b\left[\varphi \hat{Y}^R_t + \sigma \hat{C}^R_t + n_b^{-1}\left(\hat{P}^C_{a,t} - \hat{P}_{a,t}\right)\right] - (1+\varphi)\hat{Z}_{a,t}$$

or

$$\widehat{mc}_{a,t} = (\sigma+\varphi)\hat{Y}^U_t + n_b\left[\sigma \hat{C}^R_t - (\varphi\eta\Lambda_D + 1)\hat{P}^R_t + \nu \hat{C}_{a,b,t}\right] - (1+\varphi)\hat{Z}_{a,t}$$

- Degrees of stickiness are different.
  - WLOG assume country A is "sticky country"

$$\kappa_a \leq \kappa_b$$

Monetary policy

- Consider (strict) inflation targeting

$$\pi_{target,t} = \delta\pi_{a,t} + (1-\delta)\pi_{b,t} = 0$$

- SIT

$$\delta = n_a$$

- OIT

$$\delta = \delta^*$$

is optimally chosen to maximize (utilitarian Central Bank)

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[n_a U(C_{a,t}) + n_b U(C_{b,t}) - \int_0^1 V(N_{j,t}(i))di\right] \tag{1}$$

Benchmark case (Benigno 2004)

- No financial frictions and no trade frictions
- Second order approximation of (1) leads to a loss function

$$Loss = \phi_1 \mathbb{V}(\pi_{a,t}) + \phi_2 \mathbb{V}(\pi_{b,t}) + \phi_3 \mathbb{V}(\hat{Y}_t^R - \hat{Y}_t^{R,E}) + \phi_4 \mathbb{V}(\hat{Y}_t^U - \hat{Y}_t^{U,E})$$

  - Aggregate inefficiency
  - Cross country inefficiency in production
  - Within country inefficiency
- Inflation rates are relatively more important.
  - Especially, $\phi_1 > \phi_2 \gg$ other ϕ_j

Benchmark case (Benigno 2004)

$$Loss = \phi_1 \mathbb{V}(\pi_{a,t}) + \phi_2 \mathbb{V}(\pi_{b,t}) + \phi_3 \mathbb{V}(\hat{Y}_t^R - \hat{Y}_t^{R,E}) + \phi_4 \mathbb{V}(\hat{Y}_t^U - \hat{Y}_t^{U,E})$$

- Tempting to set $\pi_{a,t} = 0$ and $\pi_{b,t} = 0$.
  - But this would make relative price fixed $\rightarrow \mathbb{V}(\hat{Y}_t^R - \hat{Y}_t^{R,E})$ will be huge!

$$\begin{aligned} \hat{Y}_t^R - \hat{Y}_{t-1}^R &= -\eta\left(\pi_{a,t} - \pi_{b,t}\right) \\ &\Longrightarrow \hat{Y}_t^R = \hat{Y}_{t-1}^R \end{aligned}$$

- Let P_b float while fixing P_a → "stickiness principle":
- In other words, *OIT* is better than *SIT* as the former stabilizes the sticky country inflation better while allowing $\hat{Y}_t^R$ tracks $\hat{Y}_t^{R,E}$ with lower welfare costs
- Also, note that

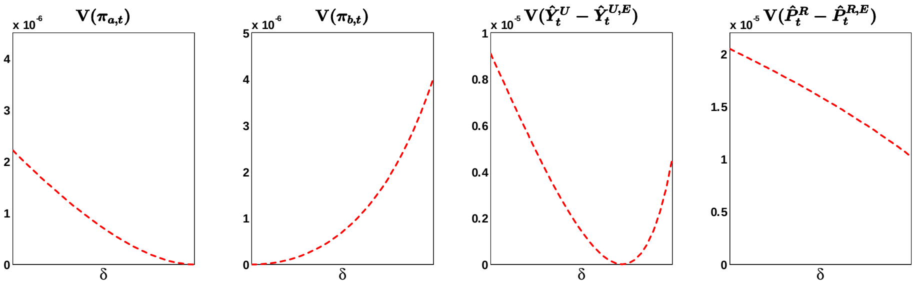
$$\begin{aligned} \hat{Y}_t^R &= \gamma_1 \hat{Y}_{t-1}^R + \gamma_2 \hat{Y}_t^{R,E} \\ \partial\gamma_1/\partial\delta &< 0 \text{ and } \partial\gamma_2/\partial\delta > 0 \end{aligned}$$

Parameterization

α_a	0.65	α_b	0.5	n_a	0.5	n_b	0.5
β	0.99	σ	2	φ	1	θ	6
η	3	$\rho_a = \rho_b$	0.96	$\sigma_{aa} = \sigma_{bb}$	0.007	σ_{ab}	0.001

- Germany: $\alpha_a = 0.65 \rightarrow 2.85Q$
- France: $\alpha_b = 0.50 \rightarrow 2.00Q$

Benchmark case



$$\text{OIT:} \quad \tilde{\pi} = 0.7\pi_a + 0.3\pi_b = 0$$

$$\text{SIT:} \quad \pi = 0.5\pi_a + 0.5\pi_b = 0$$

Financial frictions

- Add financial frictions but still no trade frictions

$$
\begin{aligned}
Loss &= \phi_1 \mathbb{V}(\pi_{a,t}) + \phi_2 \mathbb{V}(\pi_{b,t}) + \phi_3 \mathbb{V}(\hat{Y}_t^U - \hat{Y}_t^{U,E}) \\
&\quad \phi_4 \mathbb{V}(\hat{Y}_t^R - \hat{Y}_t^{R,E}) + \phi_5 \mathbb{V}\left(\hat{C}_t^R\right)
\end{aligned}
$$

- Now $\hat{C}_t^R \neq 0$ and plays two roles

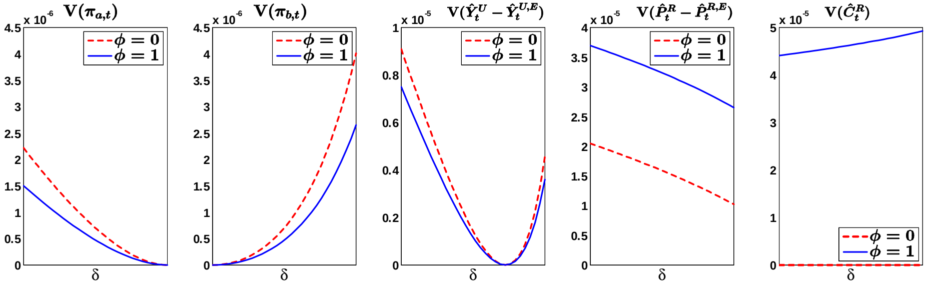
1. $\hat{C}_t^R$ is in the loss function
   - SIT is better in stabilizing $\hat{C}_t^R$ relative to OIT
2. $\hat{C}_t^R$ is in Phillips curve, which moderates $\widehat{mc}_{A,t}$ and thus $\pi_{a,t}$ – even with SIT

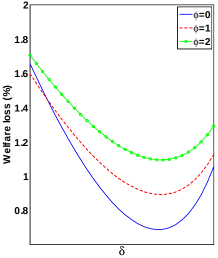
$$\pi_{a,t} = \beta \mathbb{E}_t \pi_{a,t+1} + \tilde{\kappa}_a \widehat{mc}_{a,t}$$

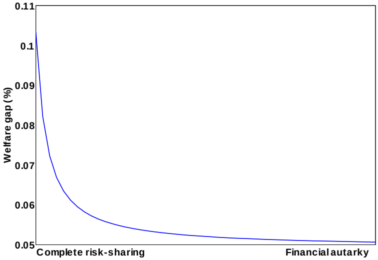
$$\widehat{mc}_{a,t} = (\sigma + \varphi)\hat{Y}_t^U + n_b \left[\sigma \hat{C}_t^R - (\varphi\eta + 1)\hat{P}_t^R\right] - (1+\varphi)\hat{Z}_{a,t}$$

   - e.g. $\hat{Z}_{a,t} \uparrow \Rightarrow \boxed{\widehat{mc}_{a,t} \downarrow} \Rightarrow P_{a,t} \downarrow \Rightarrow \hat{P}_t^R \downarrow \left(\hat{Y}_t^R \uparrow\right) \Rightarrow \hat{C}_t^R \uparrow \Rightarrow \boxed{\widehat{mc}_{a,t} \uparrow}$

- SIT is not as bad as before!


$\mathbf{V}(\pi_{a,t})$
$\mathbf{V}(\pi_{b,t})$
$\mathbf{V}(\hat{Y}_t^U - \hat{Y}_t^{U,E})$
$\mathbf{V}(\hat{P}_t^R - \hat{P}_t^{R,E})$
$\mathbf{V}(\hat{C}_t^R)$
$\phi = 0$
$\phi = 1$
δ


Welfare loss (%)
φ=0
φ=1
φ=2
δ


Welfare gap (%)
Complete risk-sharing
Financial autarky

Trade frictions

- Add trade frictions but no financial frictions

$$
\begin{aligned}
Loss &= \phi_1 \mathbb{V}(\pi_{a,t}) + \phi_2 \mathbb{V}(\pi_{b,t}) + \phi_3 \mathbb{V}(\hat{Y}_t^U - \hat{Y}_t^{U,E}) \\
& \phi_4 \mathbb{V}(\hat{Y}_t^R - \hat{Y}_t^{R,E}) + \phi_5 \mathbb{V}\left(\hat{P}_{a,t}^R - \hat{P}_{b,t}^R\right)
\end{aligned}
$$

- $\left(\hat{P}_{a,t}^R - \hat{P}_{b,t}^R\right)$ captures unsynchronized consumption bundle

$$
\widehat{\frac{C_{a,a,t}}{C_{a,b,t}}} - \widehat{\frac{C_{b,a,t}}{C_{b,b,t}}}
$$

- $(\hat{Y}_t^R - \hat{Y}_t^{R,E})$ can be decomposed

$$
\Bigg(\underbrace{\hat{Y}_t^R - \hat{Y}_t^{R,\nu=0}}_{\text{Trade friction}}\Bigg) + \Bigg(\underbrace{\hat{Y}_t^{R,\nu=0} - \hat{Y}_t^{R,E}}_{\text{Nominal friction}}\Bigg)
$$

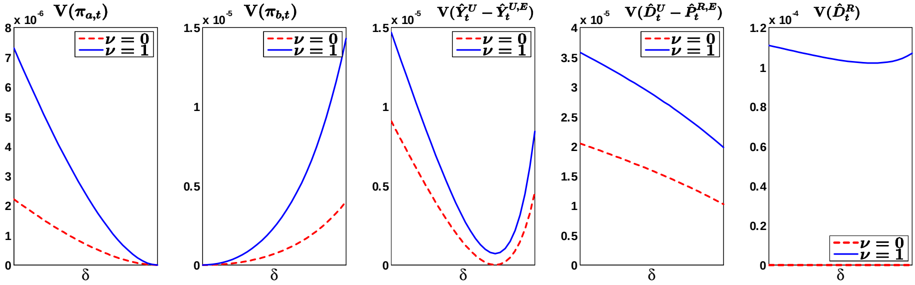
- Trade friction increase fluctuations in $\widehat{mc}_{a,t}$ and thus $\pi_{a,t}$

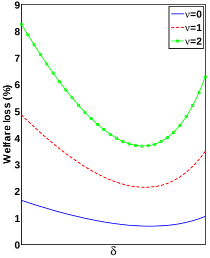
$$\pi_{a,t} = \beta \mathbb{E}_t \pi_{a,t+1} + \tilde{\kappa}_a \widehat{mc}_{a,t}$$

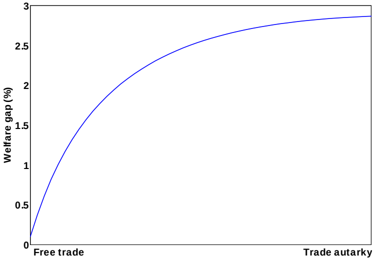
$$\widehat{mc}_{a,t} = (\sigma + \varphi)\hat{Y}_t^U + n_b \left[\begin{array}{c} \sigma \hat{C}_t^R - (\varphi\eta + 1)\hat{P}_t^R \\ +\varphi\eta(1 - \Lambda_D)\hat{P}_t^R + \nu \hat{C}_{a,b,t} \end{array} \right] - (1+\varphi)\hat{Z}_{a,t}$$

-
  - $\hat{Z}_{a,t} \uparrow \Rightarrow \boxed{\widehat{mc}_{a,t} \downarrow} \Rightarrow P_{a,t} \downarrow \Rightarrow \hat{P}_t^R \downarrow \left(\hat{Y}_t^R \uparrow\right) \Rightarrow \boxed{\widehat{mc}_{a,t} \uparrow}$
    - $\Rightarrow \hat{Y}_t^R \uparrow \searrow \Rightarrow \boxed{\widehat{mc}_{a,t} \uparrow \searrow}$ (country B imports less)
    - $\Rightarrow \hat{C}_{a,b,t} \downarrow \Rightarrow \boxed{\widehat{mc}_{a,t} \downarrow}$ (country A imports less)
  - This lowers $\widehat{mc}_{a,t}$ even further relative to the case of no trade frictions

- The benefit of OIT is pronounced.
  - SIT is even worse relative to OIT

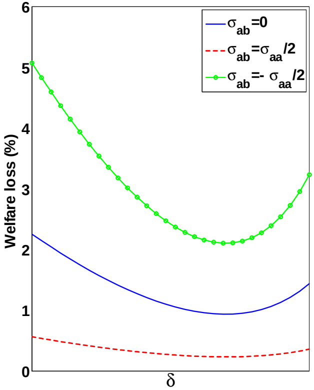

$V(\pi_{a,t})$
$V(\pi_{b,t})$
$V(\hat{Y}_t^U - \hat{Y}_t^{U,E})$
$V(\hat{D}_t^U - \hat{P}_t^{R,E})$
$V(\hat{D}_t^R)$
$\nu = 0$
$\nu = 1$
x 10^-6
x 10^-5
x 10^-4
δ


Welfare loss (%)
ν=0
ν=1
ν=2
δ

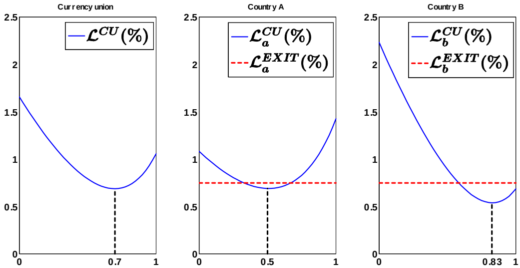

Welfare gap (%)
Free trade
Trade autarky

Shocks

$$\begin{pmatrix} \log Z_{a,t} \\ \log Z_{b,t} \end{pmatrix} = \begin{pmatrix} \rho_a & 0 \\ 0 & \rho_b \end{pmatrix} \begin{pmatrix} \log Z_{a,t-1} \\ \log Z_{b,t-1} \end{pmatrix} + \begin{pmatrix} \sigma_{aa} & \sigma_{ab} \\ \sigma_{ba} & \sigma_{bb} \end{pmatrix} \begin{pmatrix} \varepsilon_{a,t} \\ \varepsilon_{b,t} \end{pmatrix}$$



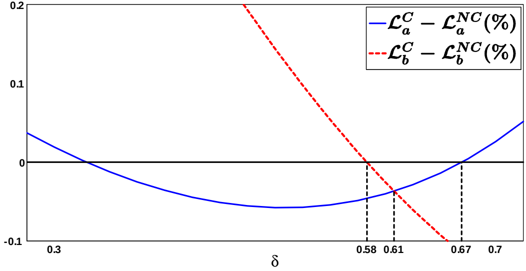
Political feasibility



- 0.7 is optimal for neither country (it's optimal for union.)
- Care more about foreign inflation (counter-intuitive.)
  - Relative demand effect
  - $V(\pi_b) \uparrow \overset{\text{precautionary}}{\longrightarrow} P_b \uparrow \rightarrow Y_a \uparrow \rightarrow N_a \uparrow$ (disutility)
  - Due to disutility from labor, (other things being equal) prefer volatile domestic inflation and stable foreign inflation.

- For some δ, countries better off exiting union.
- Constrained optimization: choose δ where

$$\begin{aligned} \mathcal{L}_a^{CU} &\leq \mathcal{L}_a^{Exit} \\ \mathcal{L}_b^{CU} &\leq \mathcal{L}_b^{Exit} \end{aligned}$$



- $\delta = 0.61$ is where "matching surpluses" are equally shared

$$\mathcal{L}_a^{Exit} - \mathcal{L}_a^{CU} = \mathcal{L}_b^{Exit} - \mathcal{L}_b^{CU}$$

Summary

- The magnitude of benefit of switching to *OIT* from *SIT* depends on the degree of union integration.
  (1) Financial integration
  (2) Real integration
  (3) Shock diffusion
  - Under some conditions, SIT may not be a bad option
    - A serious quantitative study is needed

- Politically constrained *OIT* may be different from unconstrained *OIT*.
  - Potentially closer to *SIT*.