Real Time Nowcasting of Nominal GDP under Structural Break with Credit-Card Divisia Monetary Aggregates

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This Paper

Goal

1. Construct theoretical-based monetary aggregates that measure the quantities of monetary services chosen by optimizing agents, based on utility derived from money holdings and **credit card transactions**.

2. Real time nowcasting of nominal GDP using these new Divisia monetary aggregate measures.

Motivation – Divisia Monetary Aggregates

- Problems of reliability of aggregating over monetary asset components using simple summation.
- Linear aggregation (Hicks 1946) only holds under unreasonable assumption that user-cost prices of services of individual money assets do not change over time:
  - Each asset is a perfect substitute for the others
- Simple sum aggregation is even more restrictive since it requires that coefficients of the linear aggregator function all be the same:
  - It implies that constant user-cost prices among monetary assets be exactly equal to each other. Assets must be perfect one-for-one substitutes (i.e. indistinguishable).

Motivation – Credit Card Augmented Divisia

- In reality, financial assets provide different services, and each such asset yields its own particular rate of return:
  - User costs, which measure foregone interest and thereby opportunity cost, are not constant or equal across financial assets.
  - The relative prices of U.S. monetary assets fluctuate considerably, and the interest rates paid on many monetary assets are not equal to the zero interest rate paid on currency.
- In addition, an increasing number of imperfect substitute short-term financial assets have emerged in recent decades.

This Paper – Credit Card Augmented Divisia

- Credit cards never been included in measures of money supply
  - Accounting conventions: do not permit adding liabilities, such as credit card balances, to assets, such as money.

- However, economic aggregation theory and index number theory measure service flows and based on microeconomic theory, not accounting

- This paper: new Divisia monetary aggregates that take into account the joint services of money and credit cards

Credit Card Augmented Divisia

- Credit cards: include Visa cards, Mastercards, Discover Cards, and American Express account cards providing a line of credit.

- Hard-to-obtain data from annual reports of those four sources.

- We do not include charge cards or store cards.

Model - Credit Card Augmented Divisia

$\mathbf{m}_s$ = per capita real balances of monetary assets during period s.

c_{js} = per capita real expenditure with credit card type j for transactions during period s.

z_{js} = per capita rotating real balances in credit card type j during period s from transactions in previous periods.

$y_{js} = c_{js} + z_{js}$ = per capita total balances in credit type j during period s.

r_{is} = expected nominal holding period yield on monetary asset i during period s.

Model - Credit Card Augmented Divisia

$\mathbf{x}_s$ = per capita consumptions of goods and services during period s.
$\mathbf{p}_s$ = vector of goods and services expected prices during period s.
A_s = planned per capita real holdings of the benchmark asset during period s.
R_s = expected one-period holding yield on the benchmark asset during period s.
L_s = per capita labor supply during period s.
w_s = expected wage rate during period s.

Model - Credit Card Augmented Divisia

e_{js} = expected interest rate on c_{js}.
$\bar{e}_{js}$ = expected interest rate on z_{js}.

- The Federal Reserve reports two credit card interest rates:
  - Interest rate averaged over those accounts charged interest (interest rate on z_{js}).
  - Interest rate averaged over all credit card accounts, including those accounts not being charged interest, since paid off before the end of the month. We use that measure as e_{js} , which is less than the interest rate on z_{js}
- Our model is of the representative consumer, averaged over all consumers, including those not paying interest on their credit cards

$$\max u_t = u_t(\mathbf{m}_t,\ldots,\mathbf{m}_{t+T};\mathbf{c}_t,\ldots,\mathbf{c}_{t+T};\mathbf{x}_t,\ldots,\mathbf{x}_{t+T};A_{t+T})$$
$$= U_t(v(\mathbf{m}_t,\mathbf{c}_t),v_{t+1}(\mathbf{m}_{t+1},\mathbf{c}_{t+1}),\ldots,v_{t+T}(\mathbf{m}_{t+T},\mathbf{c}_{t+T});$$
$$V(\mathbf{x}_t),V_{t+1}(\mathbf{x}_{t+1}),\ldots,V_{t+T}(\mathbf{x}_{t+T});A_{t+T})$$

subject to

$$\mathbf{p'}_s\mathbf{x}_s = \omega_s L_s + \sum_{i=1}^{n}[(1+r_{i,s-1})p^*_{s-1}m_{i,s-1} - p^*_s m_{is}]$$
$$+\sum_{j=1}^{k}[p^*_s c_{js} - (1+e_{j,s-1})p^*_{s-1}c_{j,s-1}]$$
$$+\sum_{j=1}^{k}[p^*_s z_{js} - (1+\bar{e}_{j,s-1})p^*_{s-1}z_{j,s-1}]$$
$$+[(1+\mathrm{R}_{s-1})p^*_{s-1}A_{s-1} - p^*_s A_s].$$

All purchases made at beginning of periods, and payments for purchases are either by credit cards or money. Credit card purchases repaid to the credit card company at the end of the current period or at the end of a future period, plus interest charged by the credit card company.

Intertemporal Allocation

Our model is of the representative consumer, averaged over all consumers. Since the interest rate on transactions volumes, averaged over all consumers, e_{js}, far exceeds the benchmark rate, R_t, the user cost price of credit card transactions services for the representative consumer is positive and large.

Conditional Current Intertemporal Allocation

The assumptions on homogeneous blockwise weak separability of the intertemporal utility function are sufficient for consistent two-stage budgeting. See Green (1964, theorem 4).

1. Stage 1: The consumer selects real expenditure on augmented monetary services, I_t^*, and on aggregate consumer goods for each period within the planning horizon, along with terminal benchmark asset holdings, A_{t+T}.
2. Stage 2: Augmented monetary services I_t^* are allocated over demands for the current period services of monetary assets and credit cards. That decision is to select $\mathbf{m}_t$ and $\mathbf{c}_t$ to

$$\max v(\mathbf{m}_t, \mathbf{c}_t),$$

subject to

$$\pi_t^{*\prime}\mathbf{m}_t + \tilde{\pi}_t^{*\prime}\mathbf{c}_t = I_t^*.$$

Model - Aggregation Theory

The exact quantity aggregate is the level of the indirect utility produced by the utility maximization problem:

$$\begin{aligned}\mathscr{M} &= \max\{v(\mathbf{m}_t,\mathbf{c}_t) : \pi_t'\mathbf{m}_t + \tilde{\pi}_t'\mathbf{c}_t = \mathscr{I}_t\} \\ &= max\{v(\mathbf{m}_t,\mathbf{c}_t) : \pi_t^{*\prime}\mathbf{m}_t + \tilde{\pi}_t^{*\prime}\mathbf{c}_t = \mathscr{I}_t^*\} \\ &= v(\mathbf{m}_t,\mathbf{c}_t) \\ &= \mathscr{M}(\mathbf{m}_t,\mathbf{c}_t) \\ &= \textit{augmented monetary aggregate.}\end{aligned}$$

Model - Aggregation Theory

An exact dual pair of price and quantity aggregates satisfies Fisher's factor reversal test:

$$\Pi(\pi_t, \tilde{\pi}_t) = \frac{\mathscr{I}_t}{\mathscr{M}_t}$$

Since v is linear homogeneous, it follows from Barnett (1987) that,

$$\Pi(\pi_t, \tilde{\pi}_t) = [\max_{\{\mathbf{m}_t, \mathbf{c}_t\}} \{v(\mathbf{m}_t, \mathbf{c}_t) : \pi_t' \mathbf{m}_t + \tilde{\pi}_t' \mathbf{c}_t = 1\}]^{-1}.$$

Define the cost function

$$E(v_0, \pi_t, \tilde{\pi}_t) = \min_{\{\mathbf{m}_t, \mathbf{c}_t\}} \{\pi_t' \mathbf{m}_t + \tilde{\pi}_t' \mathbf{c}_t : v(\mathbf{m}_t, \mathbf{c}_t) = v_0\}.$$

It can be proved that

$$\Pi(\pi_t,\tilde{\pi}_t) = E(1,\pi_t,\tilde{\pi}_t) = \min_{\{\mathbf{m}_t,\mathbf{c}_t\}} \{\pi_t'\mathbf{m}_t + \tilde{\pi}_t'\mathbf{c}_t : v(\mathbf{m}_t,\mathbf{c}_t) = 1\}.$$

So

$$\Pi(\pi_t,\tilde{\pi}_t) = \frac{\mathscr{I}_t}{\mathscr{M}_t} = E(1,\pi_t,\tilde{\pi}_t).$$

Resulting Quantity Aggregator Functions

In summary, we have

$$M_t = \max\{g_1(\mathbf{m}_t) : \pi_t^{*\prime}\mathbf{m}_t = \Pi_m^* M_t\}$$

and

$$C_t = \max\{g_2(\mathbf{c}_t) : \tilde{\pi}_t^{*\prime}\mathbf{c}_t = \Pi_c^* C_t\}.$$

Thus, the optimal values of the monetary and credit card quantity aggregates are related to the joint aggregate in the following manner:

$$\mathscr{M}_t = \tilde{v}(M_t, C_t).$$

Divisia Index

Discrete Time Approximations to the Divisia Index: the Törnqvist-Theil Approximation:

$$\log \mathscr{M}(\mathbf{m}_t^a) - \log \mathscr{M}(\mathbf{m}_{t-1}^a)$$

$$= \sum_{i=1}^{n} \bar{\omega}_{it}(\log m_{it} - \log m_{i,t-1}) + \sum_{i=1}^{k} \bar{\tilde{\omega}}_{it}(\log c_{it} - \log c_{i,t-1}),$$

where $\bar{\omega}_{it} = (\omega_{it} + \omega_{i,t-1})/2$, $\bar{\tilde{\omega}}_{it} = (\tilde{\omega}_{it} + \tilde{\omega}_{i,t-1})/2$,
$\omega_{it} = \pi_{it} m_{it}/(\pi_t'\mathbf{m}_t + \tilde{\pi}_t'\mathbf{c}_t)$, and $\tilde{\omega}_{it} = \tilde{\pi}_{it} c_{it}/(\pi_t'\mathbf{m}_t + \tilde{\pi}_t'\mathbf{c}_t)$.

Risk Adjustment

- Credit card interest rates are much higher than interest rates paid on monetary assets, since credit card accounts are liabilities and are unsecured. Credit cards are also subject to fraud risk. Over 80% of credit card accounts are charged interest at very high rates.

- We derive new Divisia with risk adjustment

Risk Adjustment

Existence of an Augmented Monetary Aggregate for the Consumer

We assume that the utility function, u, is blockwise weakly separable in $(\mathbf{m}_s, \mathbf{c}_s)$ and in $\mathbf{x}_s$:

$$u(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s) = F[\mathscr{M}(\mathbf{m}_s, \mathbf{c}_s), X(\mathbf{x}_s)].$$

Risk Adjustment

So Euler Equations are:

$$E_s[\frac{\partial V}{\partial m_{is}} - \rho \frac{p_s^*(R_s - r_{is})}{p_{s+1}^*} \frac{\partial V}{\partial X_{s+1}}] = 0,$$

$$E_s[\frac{\partial V}{\partial c_{js}} - \rho \frac{p_s^*(e_{js} - R_s)}{p_{s+1}^*} \frac{\partial V}{\partial X_{s+1}}] = 0,$$

and

$$E_s[\frac{\partial V}{\partial X_s} - \rho \frac{p_s^*(1 + R_s)}{p_{s+1}^*} \frac{\partial V}{\partial X_{s+1}}] = 0.$$

New Divisia with Risk Adjustment

New Generalized Augmented Divisia Index Under Risk

Definition *The contemporaneous risk-adjusted real user cost price of the services of* m_{it}^{a} *is* $\mathscr{P}_{it}^{a}$*, defined such that*

$$\mathscr{P}_{it}^{a} = \frac{\frac{\partial V}{\partial m_{it}^{a}}}{\frac{\partial V}{\partial X_t}}, i = 1,2,\ldots,n+k.$$

New Divisia with Risk Adjustment

Theorem 1(a) *The risk adjusted real user cost of the services of monetary asset i under risk is* $\mathscr{P}_{it}^{m} = \pi_{it} + \psi_{it}$, *where*

$$\pi_{it} = \frac{E_t R_t^* - E_t r_{it}^*}{1 + E_t R_t},$$

and

$$\psi_{it} = \rho\left(1 - \pi_{it}\right) \frac{Cov\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho \frac{Cov\left(r_{it}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}.$$

New Divisia with Risk Adjustment

Theorem 1(b) *The risk adjusted real user cost of the services of credit card type j under risk is $\mathscr{P}_{jt}^{c} = \tilde{\pi}_{jt} + \tilde{\psi}_{jt}$, where*

$$\tilde{\pi}_{jt} = \frac{E_t e_{jt}^{*} - E_t R_t^{*}}{1 + E_t R_t},$$

and

$$\tilde{\psi}_{jt} = \rho \frac{Cov\left(e_{jt}^{*}, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho\left(1 + \tilde{\pi}_{jt}\right) \frac{Cov\left(R_t^{*}, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}.$$

New Divisia with Risk Adjustment

Theorem 2 *In the share equations,* $\omega_{it} = \pi^a_{it} m^a_{it} / \pi^{a\prime}_t \mathbf{m}^a_t$*, we replace the user costs,* $\pi^a_t = (\pi'_t, \tilde{\pi}'_t)'$*, by the risk-adjusted user costs,* $\mathscr{P}^a_{it}$*, to produce the risk adjusted shares,* $\mathscr{S}_{it} = \mathscr{P}^a_{it} m^a_{it} / \sum_{j=1}^{n+k} \mathscr{P}^a_{jt} m^a_{jt}$*. Under our weak-separability assumption,* $V(\boldsymbol{m}_s, \boldsymbol{c}_s, X_s) = F[\mathscr{M}(\boldsymbol{m}_s, \boldsymbol{c}_s), X_s]$*, and our assumption that the monetary aggregation function* $\mathscr{M}$ *is linearly homogeneous, the following generalized augmented Divisia index holds under risk:*

$$d \log \mathscr{M}_t = \sum_{i=1}^{n+k} \mathscr{S}_{it} d \log m^a_{it}.$$

Nowcasting

Real Time Nowcasting

- Barnett, Chauvet, Leiva-Leon (BCL 2015) propose a new nowcasting mixed frequency multivariate dynamic factor model under structural break (DYMIBREAK)

- Includes all possible relevant information to produce the forecast - mixed frequency (monthly and quarterly), and ragged edge.

Real Time Nowcasting

- BCL compare predictive ability of proposed model with several univariate and multivariate specifications to nominal GDP, conditional on selection of best possible predictive variables
  - Univariate Autoregressive model
    - Linear
    - Under Structural Break
  - Multivariate models
    - Naïve model
    - Dynamic factor model mixed frequency and ragged edge
      - Linear
      - Under Structural Break

BCL Findings

- Autoregressive models and naïve model provide poor performance in nowcasting the target variable

- Small scale dynamic factor model with mixed frequency and ragged edges, using variables representing real economic activity, inflation, and monetary aggregates perform the best in nowcasting nominal GDP in real time
  - Best real-time nowcasting ability:
    - Model with - NGDP, IP, CPI and Divisia M3 or M4

This Paper –Application of New Divisia Index

Nowcasting nominal GDP with New Divisia Index

- Provides early assessments – nowcasts – of current quarterly nominal U.S. GDP growth.
- Reproduces the forecast problem of private agents and policy makers by using exact information that they have at the time predictions are made in real time.
- Evaluates accuracy of short-term forecasts using RMSE in-sample, and out-of-sample using unrevised real time data

Application of New Divisia Index

- Why Nowcasting Nominal GDP

Motivation

- Lower bound interest rate
- Fed used complementary tools to carry out monetary policy
- 2012 Annual Jackson Hole Economic Symposium. Bernanke, Woodford:
  - Quantitative Easing
  - ‘Forward Guidance’

Application

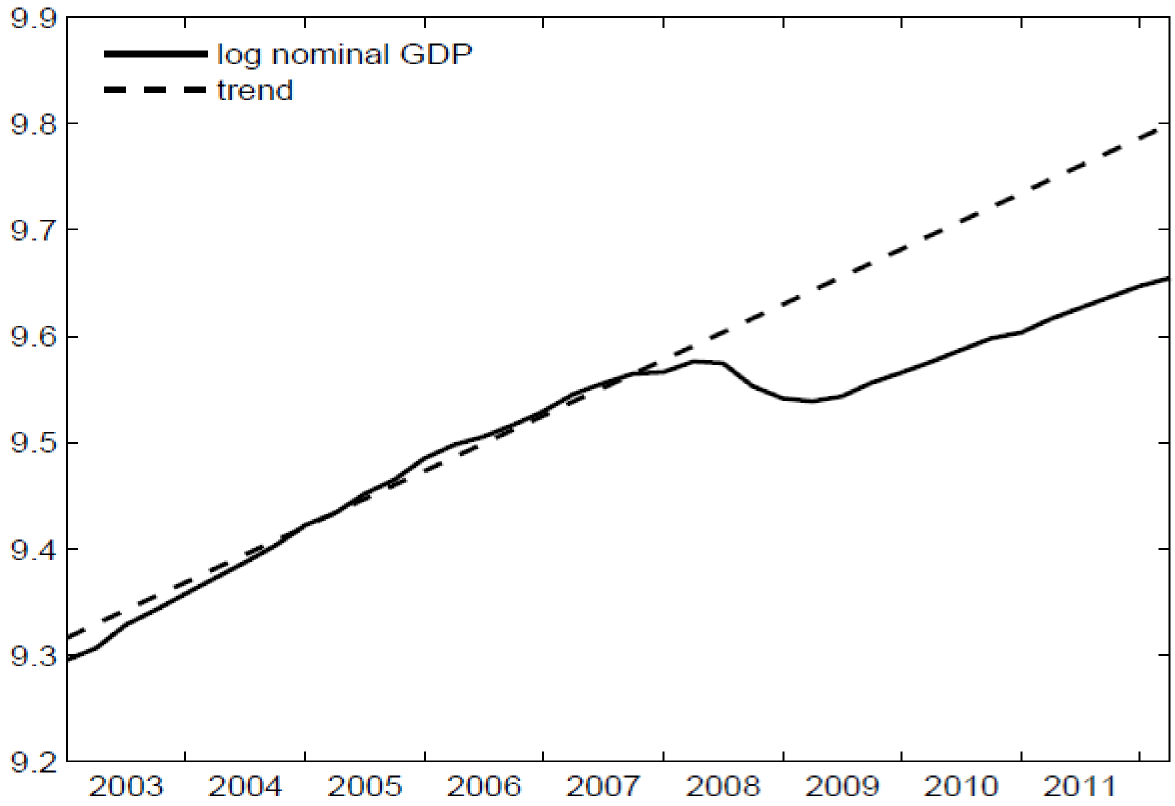
Forward Guidance

- Explicit Fed's statements about its future actions to specific developments in the economy, in addition to announcements about immediate policy actions.
- Strategy: possible changes in expectations of future economic developments that could improve the present situation, depending on the Fed target and rule
- It also facilitates *commitment* on the part of the central bank.

Application

- Woodford (2012), Romer (2011), and Hall and Mankiw (1994):
  - Fed should start targeting the path of nominal GDP, as this would constitute a powerful communication tool.

- Belongia and Ireland (2012) and Del Negro, Giannoni and Patterson (2012) – discussions and implications



Woodford (2012) – US Nominal GDP growth compared to a log-linear trend line fit to the data between 1990:Q1 and 2008:Q3 (FRED)

Reasoning for targeting GDP nominal

- Setting the objective of returning nominal GDP to its pre-crisis trajectory could improve expectations about future economic conditions.

- Such expectations would increase the incentives of households to consume more today and also firms would be more optimistic regarding their present investment decisions.

Literature Review

Dynamic Factor Models

Small Scale:
A priori selection of indicators experiencing high comovement, Stock and Watson (1991)
Nonlinear Dynamic Factor: Chauvet (1998)

Large Scale:
Large dataset associated to a target measure, Stock and Watson (1998), Forni et al. (2000)

Real-Time Nowcasting

Mixed frequency & Ragged edge: Mariano and Murasawa (2003)

Camacho and Perez-Quiros (2008) for EU,
Camacho and Perez-Quiros (2011) for Spain,
Camacho and Martinez-Marin (2013) for US

- Estimation: One step, ***Maximum Likelihood***

⇓

Our Approach to Nowcast Nominal GDP

Banbura, Giannone, Modugno and Reichlin (2010) for EU, (2012) for US.

- Estimation: Two steps, ***principal component*** & ***Kalman Smoother***, Doz, Giannone and Reichlin (2011). ***Expected Maximization***, Banbura and Modugno (2010)

Literature Review

- Literature focuses on predicting **real GDP**
- Factor models: surveys by Stock and Watson (2002), Marcellino, Stock, and Watson (2003), Wang (2009), Lombardi and Maier (2011), Winter (2011):
  - **Factor models x DSGE x VARs x Judgmental predictions:**
    - Factor model generally outperforms any other model in the short run and, often, judgmental forecasts both over the full sample and the recent crisis. Difference in forecasts statistically significant.

Multivariate Model – Naïve method

- Let Z_t be Nominal GDP, X_t be real GDP, P_t be Price Level

$$\begin{aligned} Z_t &= X_t P_t \\ \ln(Z_t) - \ln(Z_{t-1}) &= \ln(X_t) - \ln(X_{t-1}) + \ln(P_t) - \ln(P_{t-1}) \\ z_t &= x_t + p_t \end{aligned}$$

- proxy x_t and p_t, which are on quarterly frequency, with indicators with higher frequency, as Industrial Production (IP) and Consumer Price Index (CPI)

Multivariate Model – Naïve method

- MM (2003) – quarterly Z_t can be expressed into monthly time series W_t using arithmetic or geometric mean

$$Z_t = 3\left(W_t W_{t-1} W_{t-2}\right)^{1/3}$$

Taking logs to both sides and taking three period differences

$$z_t = \frac{1}{3}w_t + \frac{2}{3}w_{t-1} + w_{t-2} + \frac{2}{3}w_{t-3} + \frac{1}{3}w_{t-4}$$

Linear Multivariate Mixed Frequency Dynamic Factor Model (MFDF)

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{bmatrix} = \begin{bmatrix} \gamma_1 \left(\frac{1}{3}f_t + \frac{2}{3}f_{t-1} + f_{t-2} + \frac{2}{3}f_{t-3} + \frac{1}{3}f_{t-4}\right) \\ \gamma_2 f_t \\ \gamma_3 f_t \end{bmatrix} + \begin{bmatrix} \frac{1}{3}v_{1,t} + \frac{2}{3}v_{1,t-1} + v_{t-2} + \frac{2}{3}v_{1,t-3} + \frac{1}{3}v_{1,t-4} \\ v_{2,t} \\ v_{3,t} \end{bmatrix}$$

where γ_i are the factor loadings and $v_{i,t}$ are the associated error terms for $i = 1, 2, 3$.

- Unobserved components modeled with autoregressive dynamics:

$$\begin{aligned} f_t &= \phi_1 f_{t-1} + \phi_2 f_{t-2} + \ldots + \phi_6 f_{t-6} + e_t, \quad e_t \sim i.i.d.N(0,1) \\ v_{1,t} &= \varphi_{11} v_{1,t-1} + \varphi_{12} v_{1,t-2} + \ldots + \varphi_{16} v_{1,t-6} + \epsilon_{1,t}, \quad \epsilon_{1,t} \sim i.i.d.N(0, \sigma^2_{\epsilon_1}) \\ v_{i,t} &= \varphi_{i1} v_{i,t-1} + \varphi_{i2} v_{i,t-2} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim i.i.d.N(0, \sigma^2_{\epsilon_i}), \quad \text{for } i = 2, 3 \end{aligned}$$

Dynamic Factor (MFDF) cont.

- State Space

$$\begin{aligned} y_t &= H\beta_t + \xi_t,\ \xi_t \sim i.i.d.N(0, R) \\ \beta_t &= F\beta_{t-1} + \zeta_t,\ \zeta_t \sim i.i.d.N(0, Q) \end{aligned}$$

- MM (2003): potential missing observations substituted with a random draw v_t from a N($0, \sigma^2_v$)
  - Matrices conformable and no effect in the estimation of the model parameters

$$y_{i,t} = \begin{cases} y_{i,t} \text{ if } y_{i,t} \text{ observed} \\ \upsilon_t \text{ otherwise} \end{cases}, \quad H^*_{i,t} = \begin{cases} H_i \text{ if } y_{i,t} \text{ observed} \\ 0_{1\kappa} \text{ otherwise} \end{cases}$$

$$\xi^*_{i,t} = \begin{cases} 0 \text{ if } y_{i,t} \text{ observed} \\ \upsilon_t \text{ otherwise} \end{cases}, \quad R^*_{i,t} = \begin{cases} 0 \text{ if } y_{i,t} \text{ observed} \\ \sigma^2_\upsilon \text{ otherwise} \end{cases}$$

$$y_t = H^*_t \beta_t + \xi^*_t, \ \xi^*_t \sim i.i.d.N(0, R^*_t)$$

$$\beta_t = F\beta_{t-1} + \zeta_t, \ \zeta_t \sim i.i.d.N(0, Q)$$

- Product: Nowcast indicator of nominal GDP, f_t.

Multivariate Mixed Frequency Dynamic Factor under Break (MFDFB)

$$f_t = \mu_{S_t^m} + \phi_1 f_{t-1} + \ldots + \phi_p f_{t-P} + e_t, \quad e_t \sim i.i.d.N(0, \sigma_{S_t^v})$$

$$\mu_{S_t^m} = \mu_0(1 - S_t^m) + \mu_1 S_t^m$$

$$\sigma_{S_t^v} = \sigma_0(1 - S_t^v) + \sigma_1 S_t^v,$$

where S_t^m and S_t^v are distinct unobserved two-state Markov variables that capture permanent changes in the factor mean or variance, respectively:

$$S_t^m = 0 \text{ for } 1 \leq t \leq \tau^m \text{ and } S_t^m = 1 \text{ for } \tau^m < t \leq T - 1$$
$$S_t^v = 0 \text{ for } 1 \leq t \leq \tau^v \text{ and } S_t^v = 1 \text{ for } \tau^v < t \leq T - 1$$

Mixed Frequency Dynamic Factor Model under Break

- One time break modeled as an unknown change point, τ^k, for k = m, v, which follows constrained unobservable Markov state variables as in Chib (1998):

$$\begin{aligned}
\Pr(S_t^k &= 1|S_{t-1}^k = 1) = p_{11}^k = 1 \\
\Pr(S_t^k &= 0|S_{t-1}^k = 1) = 1 - p_{11}^k = 0 \\
\Pr(S_t^k &= 1|S_{t-1}^k = 0) = 1 - p_{00}^k \\
\Pr(S_t^k &= 0|S_{t-1}^k = 0) = p_{00}^k, 0 < p_{00}^k < 1
\end{aligned}$$

Transition matrix

$$P^k = \begin{bmatrix} p_{00}^k & 0 \\ 1 - p_{00}^k & 1 \end{bmatrix}.$$

Mixed Frequency Dynamic Factor Model under Break

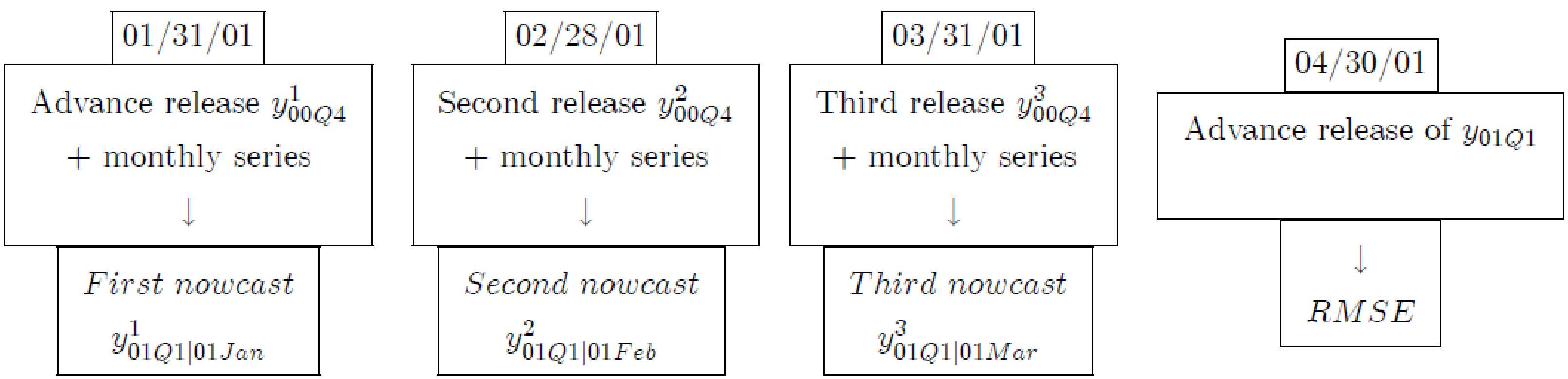
- State Space Form

$$
\begin{aligned}
y_t &= H_t^* F_t + \xi_t^*, \ \xi_t^* \sim i.i.d. N(0, R_t^*) \\
F_t &= \lambda_{S_t^m} + T F_{t-1} + \zeta_t, \ \zeta_t \sim i.i.d. N(0, Q_{S_t^v}).
\end{aligned}
$$

Timing of Forecasts

- Three main releases of NGDP for a quarter, available in the three subsequent months following that quarter: advance, second, third.

- E.g. Given NGDP last quarter of 2000, y_{00Q4}, three nowcasts of GDP first quarter of 2001, y_{01Q1}, based on three different vintages of y_{00Q4}:



Selection of Variables

- Consumer Price Index (CPI), Producer Price Index (PPI), Personal Consumption Expenditures Price Index (PCEP), and Personal Consumption Expenditure Price Index excluding Food and Energy (PCEF)

- Industrial Production (IP), Real Manufacturing and Trade Sales (MTS), Real Personal Income excluding Transfer Payments (PILTP), Nonfarm Labor (NFL)

- First, 16 possible pairwise (one real, one nominal) combinations, that will constitute our set of potential benchmark models to nowcast nominal GDP (all models include NGDP)

Three variables (NGDP and two variables):

- Best combinations :{CPI, IP}, {CPI, MTS}, {PPI, MTS}

Adding one more variable: model now contains NGDP, one real, one nominal, and one additional variable:

- The predictive performance decreases substantially

Including Other Variables: Personal Income (PI), Personal Consumption Expenditures (PCE), Average Hourly Earnings of Production and Nonsupervisory Employees (AHETPI), CFS Divisia monetary aggregates M3, M4, and M4- (which is M4 -TB), TB3, S&P500

- In most cases the performance substantially decreases as well

Best performance of larger models is in general as good or worse than smaller benchmark models:

- Best combinations {IP, CPI, M3, TBILL}, {IP, CPI, M4, TBILL}, {MTS, CPI, M3, TBILL}, {MTS, CPI, M4, TBILL}

From these results we identify a set of variables that have shown good performance across all models estimated:

IP, MTS, CPI, M3, M4 and TBILL

- 13 specifications that include different combinations of real activity, inflation, and monetary indicators that have shown the best in sample performance in fitting the target variable, nominal GDP.

- Three models: {MTS, CPI}, {IP, CPI}, {MTS, PPI}

- Six models: {IP, CPI, M3}, {IP,CPI, M4}, {IP, CPI, TBILL}, {MTS, CPI, M3}, {MTS, CPI, M4}, {MTS, CPI, TBILL}.

- Four models: IP, CPI, M3, TBILL}, {IP, CPI, M4, TBILL}, {MTS, CPI, M3, TBILL}, {MTS, CPI, M4, TBILL}

Real Time Nowcasting

- Each model estimated with data from 1967Q1 to 2000Q4, then real time nowcasts of NGDP are formed recursively for the period from 2001Q1 to 2015Q4
- E.g. the first forecast is nominal GDP growth for the first quarter of 2001, y01Q1, which is done at the beginning of February, and so on.
- RMSE associated with all predictions during are computed

Real Time Nowcasting

- All Dynamic Factor models (DFM) show substantial improvements compared to autoregressive and naïve models

- RMSE: DFM in real time are around 30% smaller than the one from AR(2) and 64% smaller than the Naïve model

Real Time Nowcasting with New Divisia Index

- Now, substituting New Divisia Indices with credit card transactions in place of Divisia Indices with only money services:

  - Nominal GDP, Industrial Production, Consumer Price Index and New Divisia Monetary Aggregate

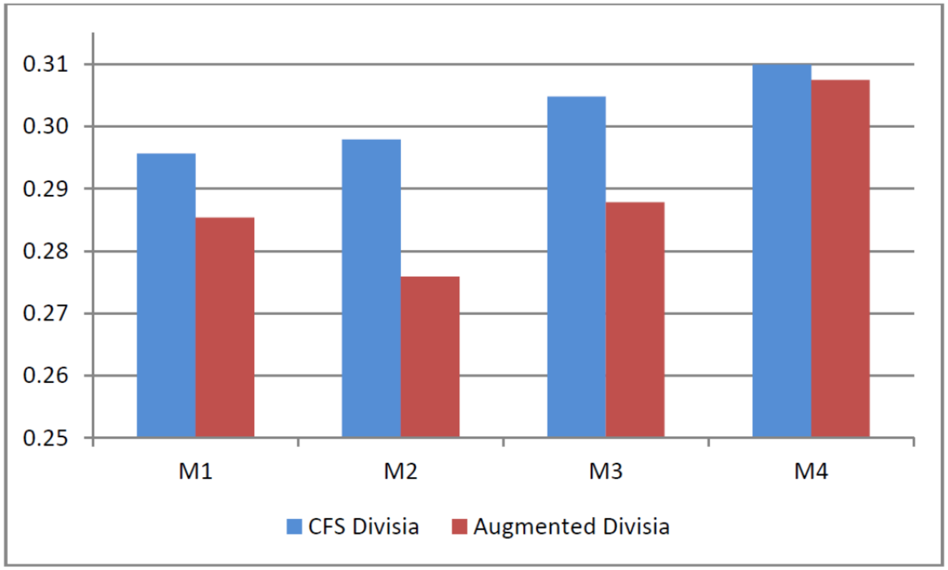
In-Sample Nowcasting

Table 1. In-sample Mean Squared Errors

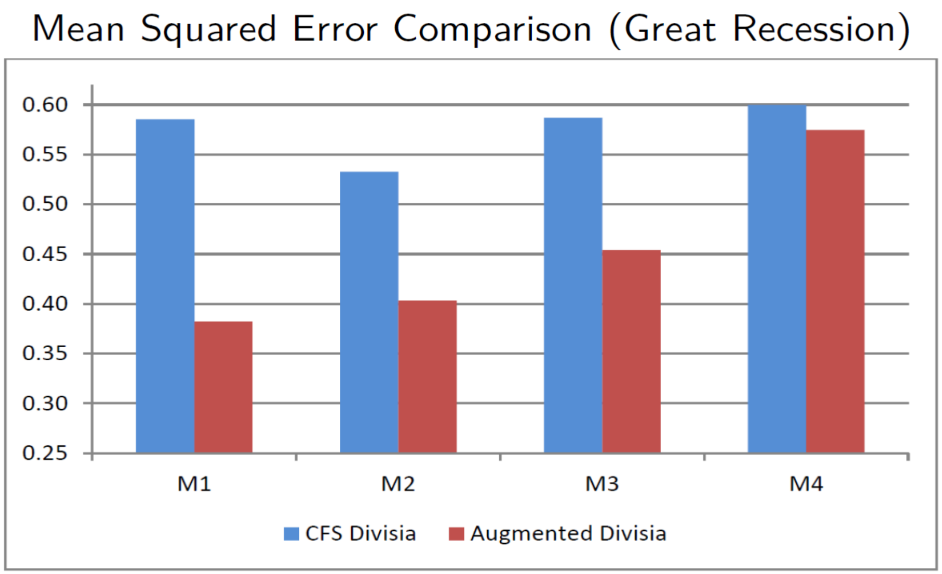
	FULL SAMPLE		GREAT RECESSION	
	CFS	Augmented	CFS	Augmented
DM1	0.16	**0.17**	0.33	**0.30**
DM2	0.18	**0.17**	0.36	**0.31**
DM3	0.16	**0.15**	0.32	**0.26**
DM4	0.18	**0.15**	0.39	**0.25**

Real Time Nowcasting

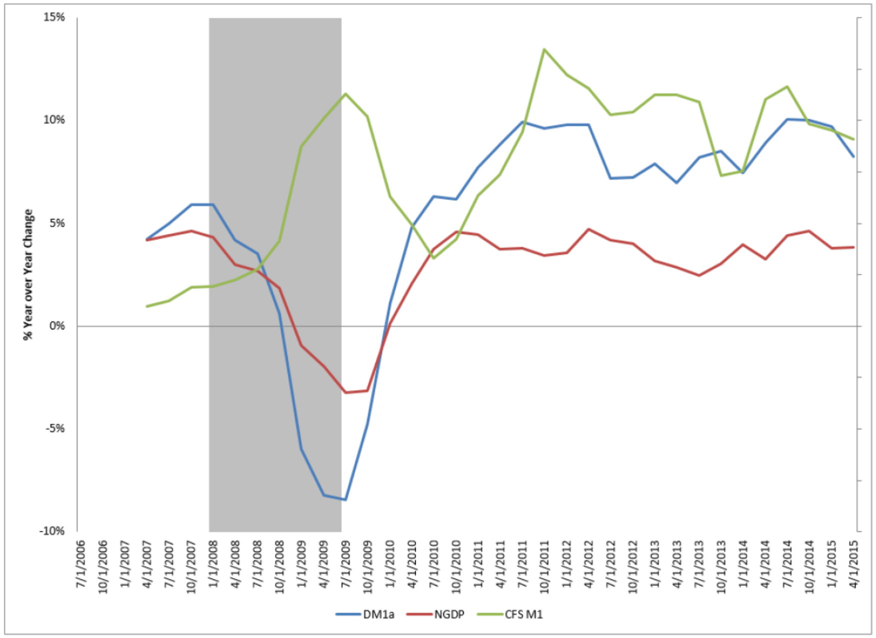
Mean Squared Error Comparison (Full Sample)



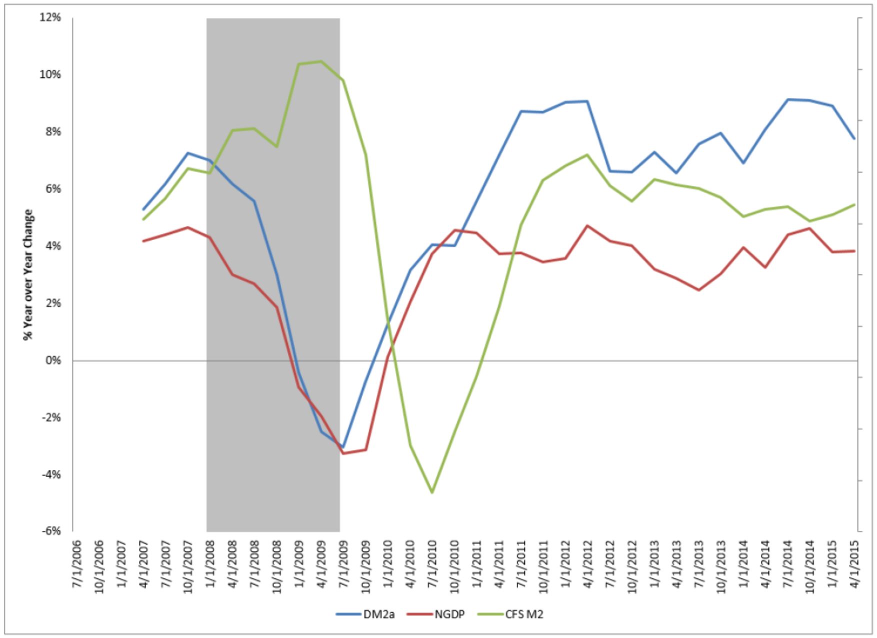
Real Time Nowcasting – Great Recession



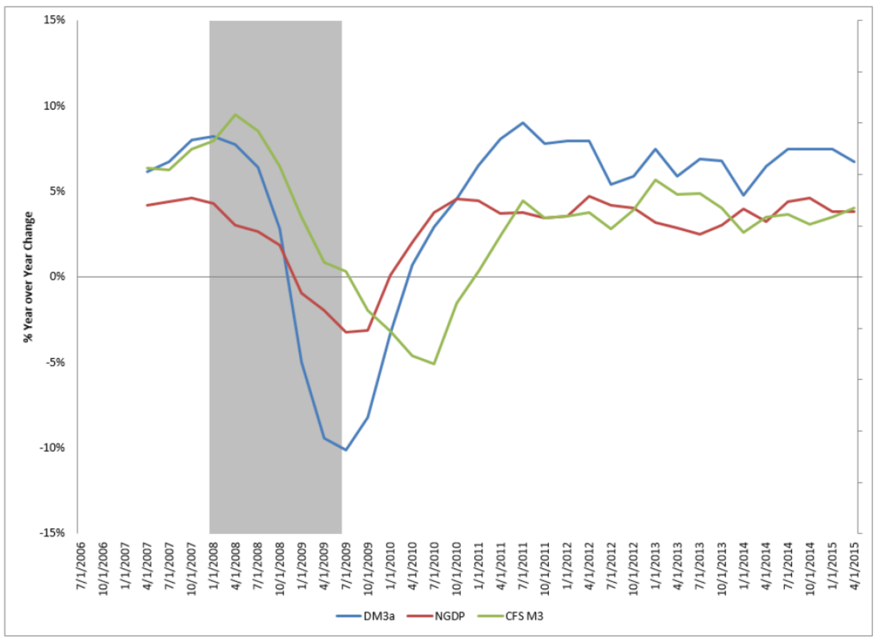
Quarterly Year-over-Year Percentage Rates of Change: M1 Level



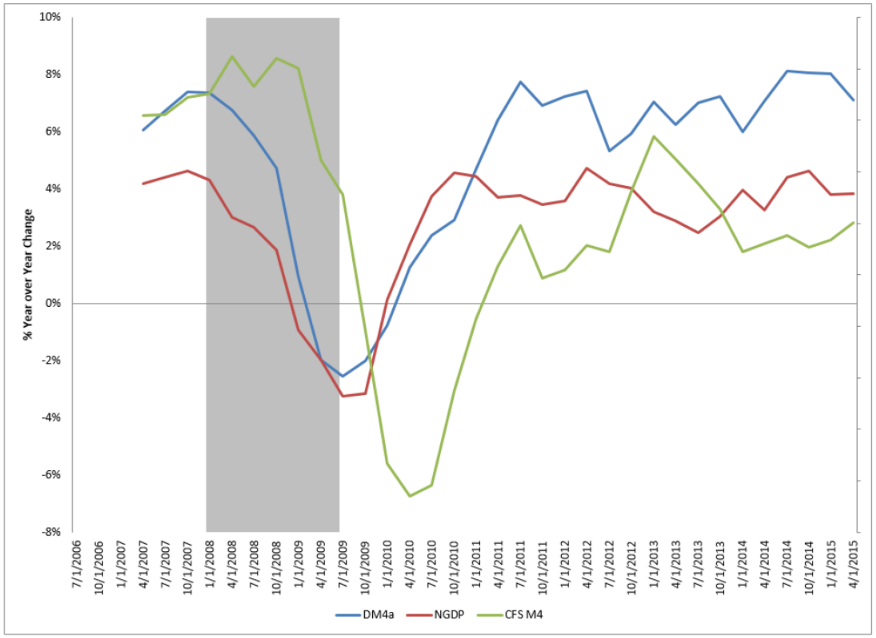
Quarterly Year-over-Year Percentage Rates of Change: M2 Level



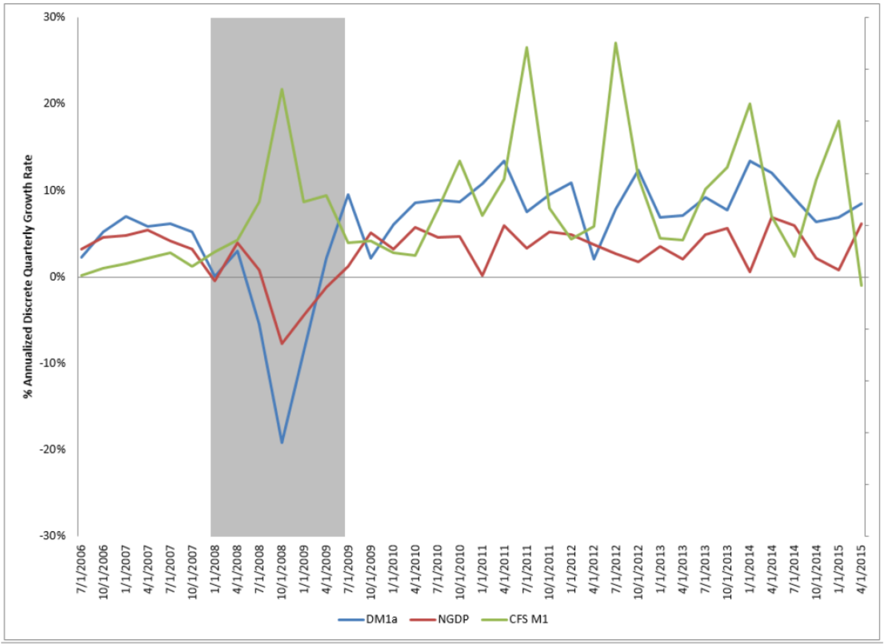
Quarterly Year-over-Year Percentage Rates of Change: M3 Level



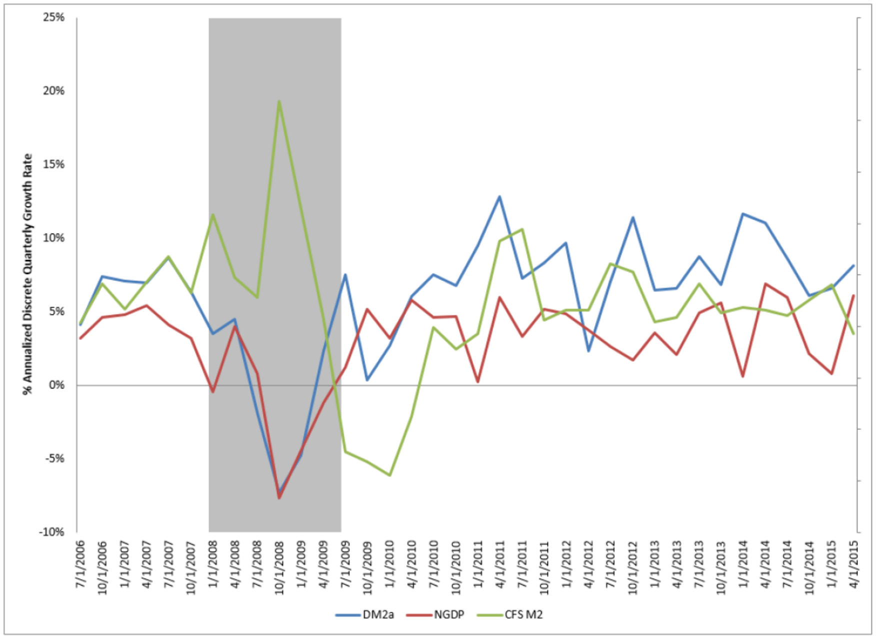
Quarterly Year-over-Year Percentage Rates of Change: M4 Level



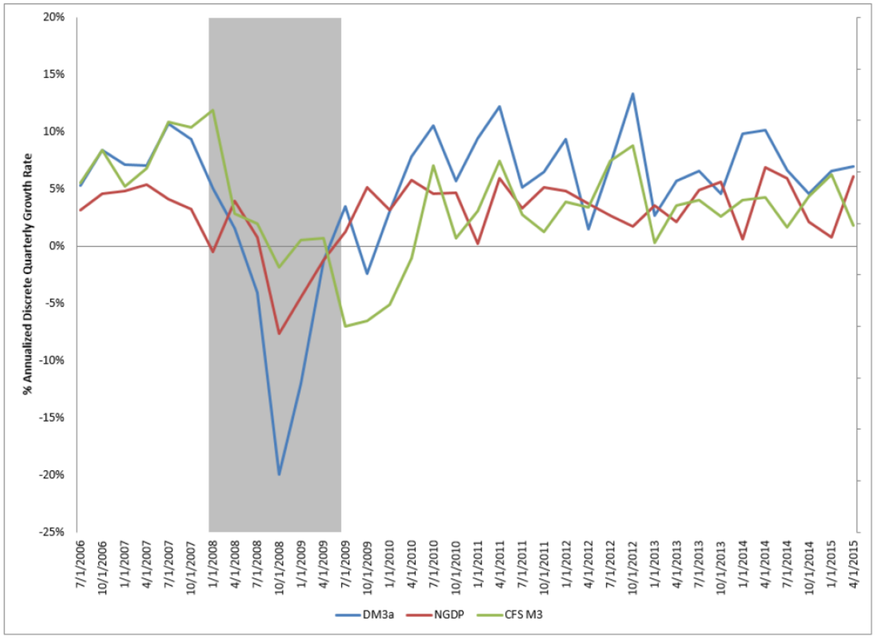
Annualized Quarterly Percentage Rates of Change: M1 Level



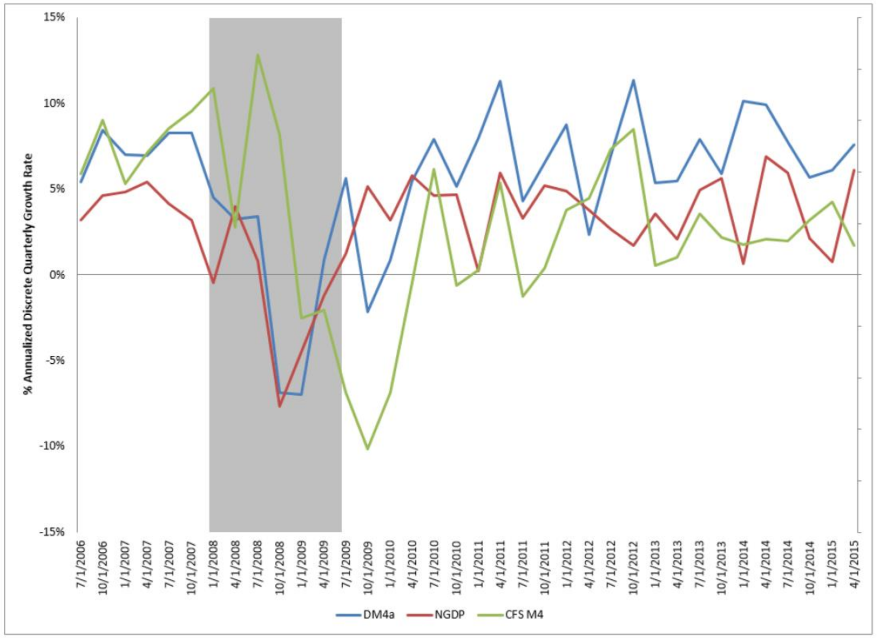
Annualized Quarterly Percentage Rates of Change: M2 Level



Annualized Quarterly Percentage Rates of Change: M3 Level



Annualized Quarterly Percentage Rates of Change: M4 Level



In the near future, the augmented Divisia monetary aggregates along with the credit cards aggregate will begin to be made available to the public monthly by the Center for Financial Stability (CFS) in New York City.

http://www.centerforfinancialstability.org/

Conclusions

- Given the non-conventional situation Fed of lower bound level of the interest rate, many economists have suggested non-conventional strategies should be adopted to decrease unemployment rate
    - One of the proposals is targeting nominal GDP.
- Effective preemptive policy requires reliable forecasts of nominal GDP. Most analyses focus on real GDP.

Conclusions

- This paper aims to help the discussions on targeting nominal GDP by providing a nowcasting model of this variable
- Autoregressive models and naïve model provide poor performance in nowcasting the target variable
- Small scale dynamic factor model with mixed frequency and ragged edges, using variables representing real economic activity, inflation, and New Divisia monetary aggregates perform the best in nowcasting nominal GDP in real time.

Conclusions and Extensions

- Economic aggregation theory and index number theory measure service flows, based on which we can include the transaction services of credit cards in monetary aggregates.
- We have implemented the theory under risk neutrality. We have further provided the extension of our theory into CCAPM risk adjustment under risk aversion.
- The current interest in nominal GDP targeting is an example of the relevancy of the Divisia monetary aggregates as indicators. Nominal GDP targeting requires the existence of monthly nominal GDP nowcasts.

Thank you