Uncertainty-driven Cooperation

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July 19, 2016
KDI

Motivation

- **Team production** is widespread (Lawler et al., 2001)
  - Theoretical literature predicts huge costs of the free-ridership problem
  - Explaining benefits: synergies, complementary skills, peer-pressure...
- **Uncertainty** in productive activities is common
- Example: a startup launching a new app
  - Uncertainty is prevalent
  - A small number of workers working on the product and the launch
  - Common form of compensation: stock options

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- Example: a startup launching a new app
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  - A small number of workers working on the product and the launch
  - Common form of compensation: stock options
- This paper: dynamic team production with uncertainty
- Main finding: **Presence of uncertainty can enhance production in teams**

Two-period Example

- Two-period repeated game
- Two agents
- In each period, each agent chooses effort $a_i \in \{L, M, H\}$
  - Cost of effort: $c(L) = 0, c(M) = c, c(H) = 2c$
  - Effort is unobservable
- Outcome in each period is either *Success* or *Fail*
  - If *Success*, each agent receives a payoff of 1
  - If *Fail*, zero payoff
- Common discount factor $\delta \leq 1$

Two-period Example

- State $\theta \in \{Good, Bad\}$
- Success probability when $\theta = Bad$:
  - Each unit of effort increases the probability by p

		2		
		H	M	L
	H	$4p$	$3p$	$2p$
1	M	$3p$	$2p$	p
	L	$2p$	p	0

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- Agent 1's stage payoff:

		2: H	2: M	2: L
1	H	$4p - 2c$	$3p - 2c$	$2p - 2c$
1	M	$3p - c$	$2p - c$	$p - c$
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- Assume $p < c < 2p$: standard free-riding effect exists
  - $p < c \Rightarrow L$ is the dominant strategy
  - $2p > c \Rightarrow H$ is the socially desirable action

Two-period Example

- State $\theta \in \{Good, Bad\}$
- Success probability when $\theta = Good$:
  - The first and second unit of effort increases the probability by p_G and p
  - Assume $p_G > p$

		2: H	2: M	2: L
1	H	$2p_G + 2p$	$2p_G + p$	$p_G + p$
1	M	$2p_G + p$	$2p_G$	p_G
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	L	$p_G + p$	p_G	0

- Assume $p_G > c$: free-riding effect exists only in the second unit of effort
  - M is the dominant strategy
  - H is the socially desirable action

Two-period Example: Equilibrium

- In the complete information case, the agents play (M, M) if $\theta = G$ and (L, L) if $\theta = B$ for all $t = 1, 2$

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- Period 2: the agents play a myopically optimal strategy
  - The agents play $\begin{cases} (M, M) & \text{if } \mu_2 > \mu^* \\ (L, L) & \text{if } \mu_2 < \mu^* \end{cases}$, where $\mu^* = \frac{c-p}{p_G - p}$

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- Period 1: the agents need to consider the effect of the current action on the future belief

Two-period Example: Equilibrium

Proposition

Suppose that $p + \delta pc > c$. Then there exist $\underline{\mu} < \mu^* < \bar{\mu}$ such that

- *There is a unique sequential equilibrium for any $\mu_1 \in (\underline{\mu}, \bar{\mu})$.*
- *In the equilibrium, the agents play (H, H) in $t = 1$.*

- Uncertainty about the state may enhance the production

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- Intuition:
  - *Success* in period 1 indicates that the state is likely to be *Good*, which leads the agents to play M in period 2
  - Knowing this, each agent has an incentive to "signal-jam" the other by increasing the success probability

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  - Knowing this, each agent has an incentive to “signal-jam” the other by increasing the success probability
- Question: Can we find a similar effect in longer-horizon team production games?

This Paper

- A tractable model of dynamic team production with uncertainty
  - Linear-Quadratic-Gaussian (LQG) framework
- Unique sequential equilibrium of the model
  - In the equilibrium, uncertainty leads to higher production
- LQG model enables us to explore several implications of the result:
  - Comparative statics *w.r.t.* team size, feedback frequency, precision of information, ...
  - Team composition: experts vs. novices in the team
  - Optimal dynamic information disclosure

Literature

- Team Production, Partnership, Free-riding in groups
  - Alchian and Demsetz (1972), Hölmstrom (1982), Radner, Myerson, and Maskin (1986)
  - Yildirim (2006), Georgiadis (2014)
- Signal-jamming effect
  - Fudenberg and Tirole (1986), Hölmstrom (1999), Cisternas (2015)
- Experimentation
  - Pure information externality: Bolton and Harris (1999), Keller, Rady, and Cripps (2005)
  - Information and payoff externality: Bonatti and Hörner (2012), Halac, Kartik, and Liu (2015), Guo and Roesler (2015)
- Repeated Games
  - Wiseman (2005, 2012), Fudenberg and Yamamato (2010)

MODEL

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- Time $t = 0, \dots, T$: discrete time, finite horizon
- Agent $i = 1, \dots, N$, common discount factor $\delta \leq 1$
- A state of the world θ is drawn from a distribution $\mathcal{N}(\mu_0, 1/h_0)$
  - θ is persistent over time
- In period t, agent i chooses unobservable effort $a_{it} \in \mathbb{R}$

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- In period t, agent i chooses unobservable effort $a_{it} \in \mathbb{R}$
- Monitoring technology:
  - Feedback (Y_t) is observed at the end of each period

$$Y_t = \theta + \sum_{i=1}^{N} a_{it} + \epsilon_t$$

where $\epsilon_t \sim \mathcal{N}(0, 1/h_\epsilon)$

Model

- Payoff:
  - Quadratic cost of effort $a_{it}^2/2$
  - Total production P is realized in period T:

$$P = \delta^{-T} \sum_{t=0}^{T} \delta^t P_t, \qquad P_t = \theta \sum_{i=1}^{N} a_{it}$$

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- The agents are risk-neutral expected utility maximizers, with agent i maximizing:

$$\begin{aligned} U &= \mathbb{E}\left[\delta^T \frac{1}{N} P - \sum_{t=0}^{T} \delta^t \frac{a_{it}^2}{2}\right] \\ &= \sum_{t=0}^{T} \delta^t \mathbb{E}\left[\frac{\theta}{N} \sum_{j=1}^{N} a_{jt} - \frac{a_{it}^2}{2}\right] \end{aligned}$$

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- Equilibrium concept: sequential equilibrium

MODEL

- Payoff is not additively separable in state and action:
  - Beliefs affect effort choices
  - "Signal-jamming" incentives present

- Feedback is additively separable in state and action:
  - All actions are equally informative
  - Variance of the posterior belief is independent of effort
  - So we can abstract from "experimentation" motives

Benchmark Cases

- Static setting $(T = 1)$:
  - The equilibrium effort level: $a_i = \mu_0 / N$
  - Effort level without free-riding: $a_i = \mu_0$

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- Complete information case $(h_0 = \infty)$:
  - The equilibrium effort level: $a_{it} = \theta/N$ for all t
  - Effort level without free-riding: $a_{it} = \theta$

EQUILIBRIUM

Belief Updating

- If the agents follow the equilibrium strategy a_{it}^*:
  - let $z_t = y_t - \sum a_{it}^*$
  - public period-t posterior $\sim \mathcal{N}(\mu_t, 1/h_t)$, where

$$\mu_t = \frac{h_{t-1}\mu_{t-1} + h_\epsilon z_{t-1}}{h_{t-1} + h_\epsilon}, \qquad h_t = h_{t-1} + h_\epsilon$$

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- If agent i deviates to $a_{i,t-1} = a_{i,t-1}^* + \alpha$:
  - public period-t posterior $\sim \mathcal{N}(\mu_t, 1/h_t)$

$$\mu_t = \frac{h_{t-1}\mu_{t-1} + h_\epsilon(\theta + (a_{i,t-1} + \sum_{i \neq j} a_{i,t-1}^*) - \sum a_{i,t-1}^*)}{h_{t-1} + h_\epsilon}$$

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  - agent i's (private) period-t posterior $\sim \mathcal{N}(\tilde{\mu}_{it}, 1/h_t)$, where

$$\tilde{\mu}_{it} = \frac{h_{t-1}\mu_{t-1} + h_\epsilon(z_{t-1} - \alpha)}{h_{t-1} + h_\epsilon} = \mu_t - \frac{h_\epsilon}{h_t}\alpha$$

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  - public period -$(t+k)$ posterior:

$$\mu_{t+k} = \tilde{\mu}_{i,t+k} + \rho_{t+k}\alpha,$$

where $\rho_s = h_\epsilon / h_s$: rate at which the deviation in period $t < s$ affects μ_s.

Towards Equilibrium: Backward Induction

- Let μ_t and $\tilde{\mu}_{it}$ be the public and agent i's private period-t posterior mean
    - on the equilibrium path, $\tilde{\mu}_{it} = \mu_t$ for all i
- Since $c(a) = a^2/2$, optimal action=marginal benefit of effort

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- Period T:

$$a^*_{iT} = \frac{\tilde{\mu}_{iT}}{N}.$$

  - let $\xi_T = 1/N$: **Belief sensitivity of effort** in period T

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- Period $T-1$:

$$a^*_{i,T-1} = \frac{\tilde{\mu}_{i,T-1}}{N}(1 + \underbrace{\delta(N-1)\rho_T\xi_T}_{\text{signal-jamming}}) \equiv \xi_{T-1}\tilde{\mu}_{i,T-1}$$

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- Marginal benefit of effort does <u>not</u> depend on the other agents' beliefs

Towards Equilibrium: Backward Induction

- Period $T-2$:
- Consider a deviation at $t=T-2$: $a_{i,T-2} = a^*_{i,T-2} + \alpha$ $(\alpha > 0)$

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$$\mu_{T-1}=\tilde{\mu}_{i,T-1}+\rho_{T-1}\alpha$$

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$$\mu_T = \tilde{\mu}_{i,T} + \rho_T(1 - \underbrace{\rho_{T-1}\xi_{T-1})\alpha}_{\text{due to } a_{i,T-1}}$$

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- Then the marginal benefit of effort at $t = T-2$ is

$$a^*_{i,T-2} = \frac{\tilde{\mu}_{i,T-2}}{N}(1 + (N-1)[\delta\rho_{T-1}\xi_{T-1} + \delta^2\rho_T\xi_T(1 - \rho_{T-1}\xi_{T-1})]$$

Equilibrium

Proposition

There exists a unique sequential equilibrium. In the equilibrium,

$$a_{it}^{*} = \xi_t \tilde{\mu}_{it},$$

where

$$\xi_t = \frac{1}{N} + \frac{N-1}{N} \sum_{k=t+1}^{T} \left[\delta^{k-t} \rho_k \xi_k \prod_{s=t+1}^{k-1} (1 - \rho_s \xi_s) \right],$$

and $\xi_T = \frac{1}{N}$.

- The agents' actions are only a function of their private posterior mean and calendar time

EQUILIBRIUM: GRAPH

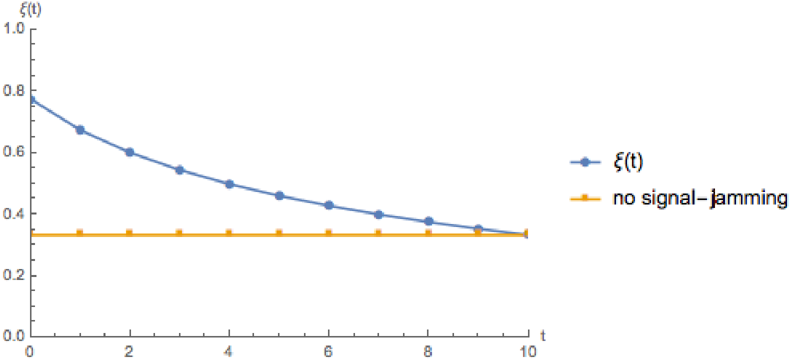

FIGURE: Equilibrium value of ξ_t $(N = 3, T = 10, \delta = 0.98, h_0 = h_\epsilon = 1)$

- Signal-jamming incentives are larger earlier: As t increases,
  - there are fewer periods until the end
  - the belief becomes more precise

Perfect Monitoring Case

- If each agent's effort level is observable to others, then there is no signal-jamming effect

Proposition

In the perfect monitoring case, there exists a unique Nash equilibrium where for any $t = 0, \ldots, T$,

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- Imperfect monitoring is necessary for the signal-jamming effect
- Alchian and Demsetz (1972):

> *... In team production, marginal products of cooperative team members are not so directly and separably (i.e., cheaply) observable. ... The costs of metering or ascertaining the marginal products of the team's members is what calls forth new organizations and procedures.*

Continuous-Time Limit

- Real-time horizon $\Gamma > 0$, let Δ be the length of a period
  - $r > 0$: discount rate $\Rightarrow \delta = e^{-r\Delta}$
  - $\eta > 0$: information disclosure rate $\Rightarrow h_\epsilon = \eta\Delta$
  - per-period cost $= \frac{a_{it}^2}{2}\Delta$
  - per-period production $= \theta \sum_i a_{it}\Delta$
- We consider $\Delta \to 0$:

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- Belief updating:

$$d\mu_t = \frac{\eta(z_t - \mu_t)}{h_t} = \frac{\eta}{h_t}dW_t \qquad \dot{h}_t = \eta$$

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- Equilibrium sensitivity ξ_t follows

$$\dot{\xi}_t = \underbrace{r\left(\xi_t - \frac{1}{N}\right)}_{\text{discounting}} - \underbrace{\frac{\eta}{h_0 + \eta t}\xi_t\left(1 - \xi_t\right)}_{\text{signal jamming}}$$

and $\xi_T = 1/N$.

CONTINUOUS-TIME LIMIT: GRAPH

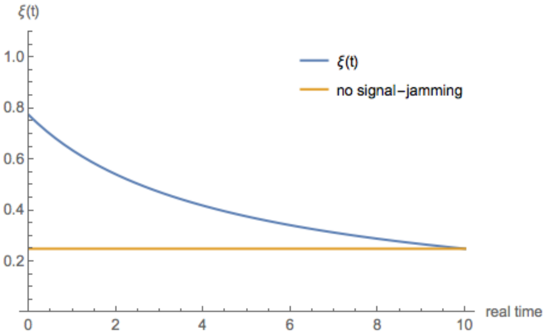

FIGURE: Equilibrium sensitivity $\xi(t)$ $(N = 4, T = 10, \eta = 2, h_0 = 3, r = 0.001)$

- Our result extends to the continuous-time limit.

Comparative Statics

Proposition

(1) Individual effort $(\xi(t))$ decreases in r.
(2) Individual effort $(\xi(t))$ decreases in h_0 and increases in η.

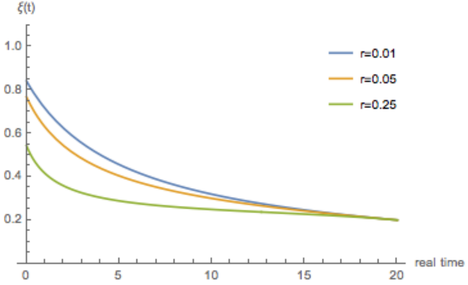

Figure: Effect of r

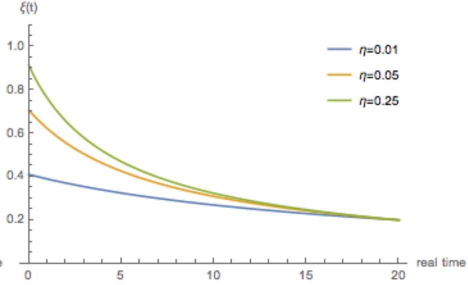

Figure: Effect of η

Individual effort over time $(N = 5, T = 20)$

Comparative Statics: Team Size

Proposition

Individual effort ($\xi(t)$) decreases in team size but total effort ($N \times \xi(t)$) increases in team size.

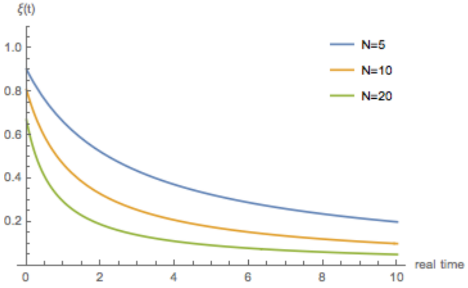

FIGURE: Graph of $\xi(t)$

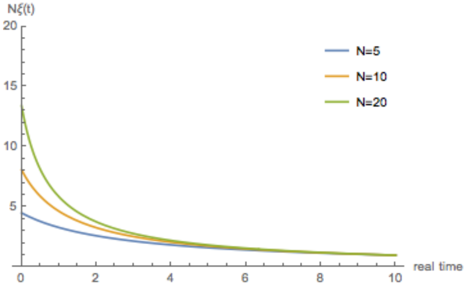

FIGURE: Graph of $N \times \xi(t)$

Effect of team size ($N = 5, 10, 20$)
($r = 0.003, T = 10, h_0 = 0.8, \eta = 4$),

Comparative Statics: Effect of Period Length

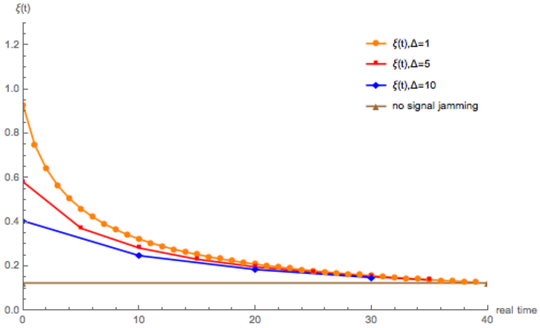

FIGURE: Belief sensitivity $\xi(t)$ for different period lengths$(N = 8, r = 0.01, T = 40, h_0 = \eta = 1)$

- Effort increases as Δ gets smaller
  - Abreu, Milgrom, Pearce (1991), Sannikov and Skrzypacz (2007): scope of cooperation limited as $\Delta \to 0$

Extensions

- Consider the following two generalizations of the model:

1. The period-t feedback y_t is given by

$$y_t = \kappa_\theta \theta + \kappa_a \sum a_{i,t} + \epsilon_t.$$

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- Then in the continuous-time limit, equilibrium ξ_t follows

$$\dot{\xi}_t = r\left(\xi_t - \frac{k}{N}\right) - \frac{\eta \kappa_\theta \kappa_a}{h_0 + \eta t \kappa_\theta^2} \xi_t \left(k - \xi_t\right)$$

EXTENSIONS: LIMIT RESULT

- When either $\kappa_a \to \infty$, $\kappa_\theta \to 0$, or $k \to \infty$, the agents' equilibrium effort converges to the first-best level

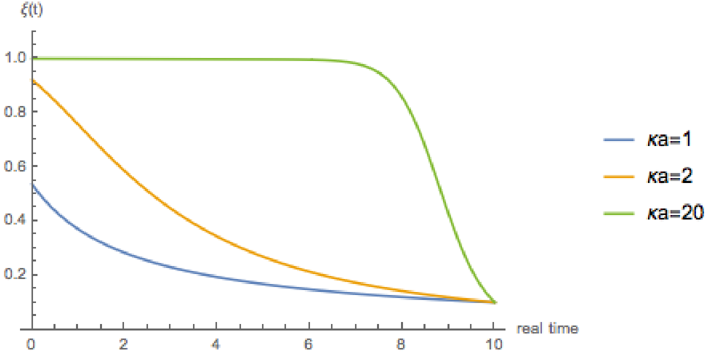

FIGURE: Equilibrium sensitivity ξ_t $(N = 10, r = 0.01, \eta = h_0 = 1, , \kappa_\theta = k = 1)$

INFINITE-HORIZON GAME

Infinite-horizon Game: Existence of MPE

- The benchmark model can be extended to the infinite-horizon $(t = 0, \ldots)$
  - Reinterpret δ as the probability of the project survival

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  - Reinterpret δ as the probability of the project survival

Proposition

There exists a Markov perfect equilibrium of the infinite-horizon game.

- As $T \to \infty$, the signal-jamming effect is robust
- Furthermore, ξ_t converges to a finite number as $T \to \infty$: the signal-jamming incentive does not blow up
- The results in the limit of finite-horizon model and the infinite-horizon model agree

Comparison to Trigger Strategies

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- Conjecture in our model: as $\Delta \to 0$, the maximum equilibrium payoff converges to that of our equilibrium
  - Our equilibrium does not rely on trigger strategies, yet the agents exert high effort.

Asymmetric Information

Asymmetric Information

- The members in a team may have difference information about the state
- We look an alternative model in which the agent can be either an expert or a novice
  - The experts perfectly know the true state, while the novices are uninformed
- Question:
  - How does the expert reveal the information over time?
  - Does having an expert increase the total production?

Asymmetric Information: Model

- Continuous time $t \in [0, T]$
- Suppose there are N^e experts and N^n novices in the team
  - The experts perfectly know θ
  - The novices have a common prior belief $\sim \mathcal{N}(\mu_0, 1/h_0)$
- The signal $\{Y_t\}$ follows a stochastic process

$$dY_t = (\theta + \sum_i a_{it})dt + \frac{1}{\sqrt{\eta}} dW_t$$

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- We focus on equilibria in linear Markov strategies
  - The expert's strategy: $a_t^e = \gamma_t \theta + \psi_t$
  - The novice's strategy: $a_t^n = \xi_t \mu_{it}$
  - Insider trading models (Kyle, 1985; Back, 1992), Cournot competition (Bonatti, Cisternas, and Toikka, 2015)

Asymmetric Information: Equilibrium

Theorem

There exists a symmetric linear Markov perfect equilibrium, in which the coefficients of the strategies are obtained from the solution to following system of differential equations

$$\dot{\gamma}_t = \frac{(1+N^e\gamma_t)(\gamma_t - \frac{1}{N})rh_t - \eta(1+\gamma_t N^e)\frac{N^n}{N}\xi_t}{(1+\frac{N^e}{N})h_t}$$

$$\dot{\xi}_t = \left(r - \frac{N-N^e-2}{N}\frac{\eta}{h_t} + (\xi_t - \frac{1}{N})\right)\left(\xi_t - \frac{1}{N}\right) - \frac{N-N^e-1}{N^2}\frac{\eta}{h_t}$$

$$\dot{h}_t = \eta(1+N^e\gamma_t)^2$$

and boundary conditions $\gamma_T = \xi_T = \frac{1}{N}$.

- Solution: Hamilton-Jacobi-Bellman equation with a quadratic value function ⇒ Boundary Value problem

EQUILIBRIUM STRATEGY: GRAPH

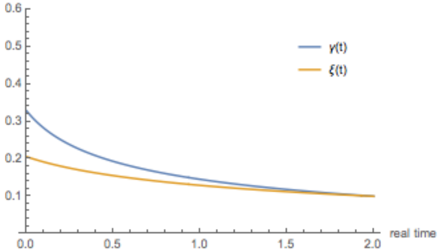

FIGURE: Equilibrium strategy of the expert (γ_t) and the novices (ξ_t) $(N^e = 1, N^n = 9, T = 2)$

- Expert: γ_t is decreasing over time
  - The expert gradually reveal his private information
- Novice: $\xi_t > 1/N$: incentive to manipulate the other novices' beliefs

Applications & Future Work

Dynamic Information Disclosure Design

- Consider a principal who hires a group of agents working in teams
  - He can control the provision of information in each period ($h_\epsilon(t)$)
  - He wants to maximize the total production

Conjecture

Assume that δ is sufficiently large. Then in the optimal information design scheme, the signal precision increases over time.

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- "Backloading information" is optimal
  - Higher $h_\epsilon(t) \Rightarrow$ higher incentive in period t, but lower incentive for all other periods
  - Incentive is accumulated in backward direction:
    - Higher sensitivity in the future $\Rightarrow$ More incentive to effort now

Contracting

- Consider a (potentially asymmetric) share structure $s = (s_1, \ldots, s_N)$ which specifies the fraction of the output owned by each agent

$$s \in [0,1]^N, \qquad \sum_{i=1}^{N} s_i = 1$$

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  $\Rightarrow$ incentives to be maximized under the equal share
- Question: optimal share structure with heterogeneous agents

Conclusion

- We present a new mechanism to induce cooperation in team production with uncertainty about the project state
- Our mechanism has several distinctive properties:
  - The mechanism also works in the finite-horizon model and the continuous-time limit
  - Inability to observe the other's effort is necessary
  - Frequent feedback increases the effort level
- Future work:
  - Stochastic states
  - A model with private monitoring
  - More comparison with the trigger mechanism
    - Implications to organization economics
  - Public good model

Literature

- Team Production, Partnership, Free-riding in groups
  - Alchian and Demsetz (1972), Hölmstrom (1982), Radner, Myerson, and Maskin (1986)
  - Georgiadis (2014)
- Signal-jamming effect
  - Fudenberg and Tirole (1986), Hölmstrom (1999), Cisternas (2015)
- Experimentation
  - Pure information externality: Bolton and Harris (1999), Keller, Rady, and Cripps (2005)
  - With payoff externality: Bonatti and Hörner (2012), Halac, Kartik, and Liu (2015), Guo and Roesler (2015)
- Repeated Games
  - Wiseman (2005, 2012), Fudenberg and Yamamato (2010)

Stochastic State: First-order Condition

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Stochastic State: Dynamic Programming

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