Liquidity Risk and Time-Varying Correlation Between Equity and Currency Returns

Kuk Mo Jung

School of Economics, Henan University

Korea Development Institute

December 16, 2015

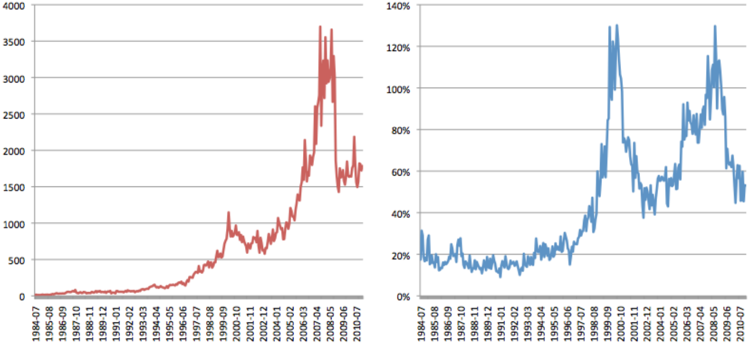
Motivation

- ✓ Difficult to tie FX rates to macroeconomic fundamentals
  - Uncovered Interest Rate Parity (UIP) puzzle
  - Meese and Rogoff (1983) puzzle
- ✓ Recent FX determination theories revolve around non-macro based models
  - FX market microstructure literature; e.g., Lyons (2001), Evans (2010), and etc.
- ✓ Another strand of recent literature studies a link between equity and FX markets
  - Due to a development in solving DSGE models of international portfolio choice: e.g., Devereux and Sutherland (2012); Tille and Van Wincoop (2008)
  - Due to rapidly increasing cross-border equity transactions

Motivation

Figure: International Equity Transaction Trend

Gross cross-border transactions in equity for the U.S. | Ratio of cross-border equity transactions relative to bond transactions



Source: Treasury International Capital System, U.S. Treasury

A link between equity and FX returns?

- ✓ In general, a negative relationship between relative equity and FX returns was found
  - High equity return currencies tend to **depreciate** at short to medium frequencies
  - Hau and Rey (2006), Cappiello and Santis (2007), and Melvin and Prins (2015)
- ✓ Another studies, though rare, argue that the relationship can be non-negative in theory
  - Pavlova and Rigobon (2007) and Cenedese, Payne, Sarno, and Valente (2015)
- ✓ However, none of them studies if the relationship exhibits **time-variation**
  - Important for the nature and magnitude of risk sharing in international markets
  - The UIP also exhibits time-variation; Brunnermeier, Nagel, and Pedersen (2006)

What I do and found

The objective of this paper is twofold:

1. Empirically re-examine the relationship in real terms using updated data set
    - Do high equity return currencies always depreciate?

      Answer: No
    - If not, when and under what condition does the relationship overturn its sign?

      Answer: The negative correlation exhibits strong tendency to overturn its sign when the measure of market volatility soars

2. Offer one new and plausible theory of FX determination for these findings
    - Long run risks model of Bansal and Shaliastovich (2013) type augmented with time-varying equity market liquidity
    - Can match empirical evidence both qualitatively and quantitatively

Empirical evidence: data and methodology

Real monthly stock market returns between a month t and $t+1$:

$$R_t^i = \ln(SI_{t+1}^i) - \ln(SI_t^i) - \left\{\ln(CPI_{t+1}^i) - \ln(CPI_t^i)\right\}$$

- ✓ SI_t^i is a stock market index at a month t for a country i: From St.Louis FED
- ✓ CPI_t^i is the CPI for a country i at a month t: From St.Louis FED

Change in *real* FX rates of the country i from t to $t+1$, i.e., Δq_t^i:

$$\Delta q_t^i = \left\{\ln(FX_{t+1}^i) - \ln(FX_t^i)\right\} + \left\{\ln(CPI_{t+1}^i) - \ln(CPI_t^i)\right\} - \left\{\ln(CPI_{t+1}^{U.S}) - \ln(CPI_t^{U.S})\right\}$$

- ✓ FX_t^i is the nominal U.S. dollar price per unit of the country i's currency, i.e., $\$/¥$, at a month t: From New York FED

Empirical evidence: data and methodology

Amihud (2002) measure of stock market illiquidity:

$$lq_t^i = \frac{|R_t^i|}{TV_t^i},$$

✓ TV_t^i is a measure for aggregate stock market trade volume for a country i at a month t: From Yahoo Finance

Stock market liquidity volatility for a country i at a month t:

$$\sigma^2_{lq_t^i}$$

✓ the volatility of lq^i at a month t, calculated using a 2-year rolling variance measures

Short-run economic uncertainty index for a pair of {county i and U.S.}:

$$X_t^i = \sigma^2_{R_t^i} + \sigma^2_{R_t^{U.S}}$$

✓ $\sigma^2_{R_t^i}$: the volatility of R^i at a month t, calculated using a 2-year rolling variance measures

Empirical evidence: basic results

Table: $\Delta q_t^i = \alpha_i + \beta[R_t^i - R_t^{U.S}] + \varepsilon_t^i$

Periods	1991/01-1998/12	1999/01-2001/12	2002/01-2010/12	2011/01-2014/12
Panel with FE	-0.1710378^{***}	1.220263^{**}	-0.190125^{***}	-0.0347795
Pooled OLS	-0.1757183^{***}	1.158778^{**}	-0.1868499^{***}	-0.0371741
# of cross-section	19	19	19	19
# of periods	95	36	108	48
# of observations	1805	684	2052	912

Note: *, ** and *** indicates that the coefficient is significant at 10%, 5% and 1% level respectively.

Table: $\Delta q_t = \alpha + \beta[R_t - R_{US,t}] + \varepsilon_t$

Countries	$\hat{\beta}$
France	0.0575972
Germany	0.1842392
Italy	0.2453416^{**}
Spain	0.1219099

Note: The sample periods is for European Debt Crisis (2011/01-2012/12).

The same significance level applies to *, ** and *** as before.

Empirical evidence: conditional effects of uncertainty

Table: $\Delta q_t^i = \alpha_i + \beta[R_t^i - R_t^{U.S}] + \gamma[R_t^i - R_t^{U.S}]X_t^i + \varepsilon_t^i$

Methods	Panel with FE		Pooled OLS	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
	-0.1680585	9.937949*	-0.175161	9.976492*
# of cross-section	19		19	
# of periods	264		264	
# of observation	5016		5016	

Note: The same significance level applies to *, ** and *** as before.

Empirical evidence: conditional effects of uncertainty

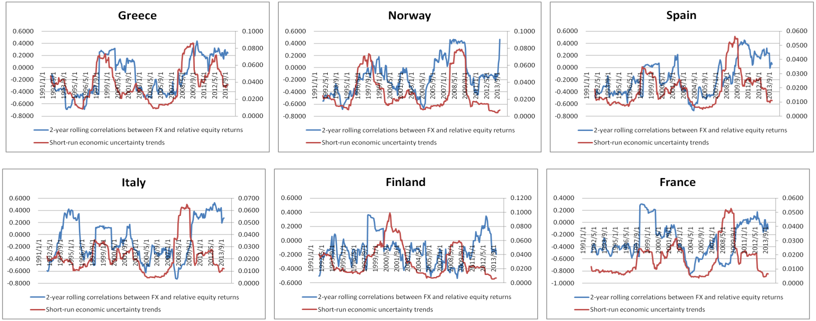
Table: $\Delta q_t = \alpha + \beta[R_t - R_{US,t}] + \gamma[R_t - R_{US,t}]X_t + \varepsilon_t$

Countries	$\hat{\beta}$	$\hat{\gamma}$
Austria	-0.6285739^{*}	6.551034
Belgium	0.0730233	15.85387
Finland	-0.0218074	3.115712
France	-0.2578065	30.51101
Germany	-0.250619^{*}	4.238589
Greece	-1.46096^{**}	46.47461^{***}
Ireland	0.9100167	16.04203
Italy	-0.8357776	66.95506
Japan	0.0148532	-7.140679^{*}
Netherlands	-0.5183074^{***}	8.739108
Norway	-0.268392^{***}	5.075721^{***}
Portugal	-0.4271297	44.80914
South Korea	-0.1388388	3.423984
Spain	-0.4306854	27.5968
U.K.	-0.4982304^{**}	-11.54311

Note: The same significance level applies to *, ** and *** as before.

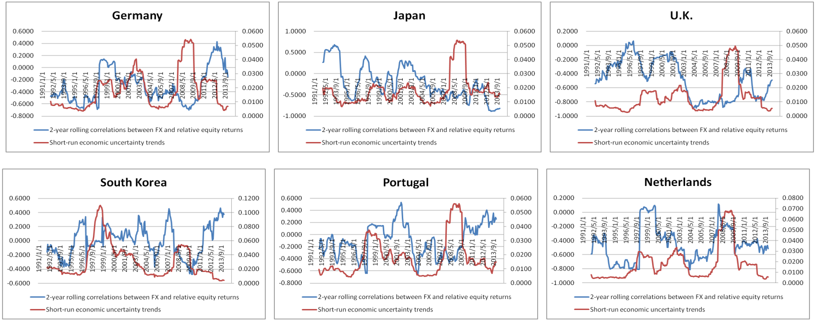
Empirical evidence: time-varying rolling correlations between Δq_t^i and $[R_t^i - R_t^{U.S}]$

Figure: Correlation between FX and equity returns with a rolling window of 2-year periods



Empirical evidence: time-varying rolling correlations between Δq_t^i and $[R_t^i - R_t^{U.S}]$

Figure: Correlation between FX and equity returns with a rolling window of 2-year periods



How I explain this evidence

- ✓ Potentially several factors can create time-varying consequences for pricing kernels
  - e.g., incomplete international asset markets, flight to quality, unconventional monetary policy, ambiguity aversion, rational inattention, robust control, and etc.
- ✓ A key is to have a volatility or uncertainty driven pricing kernel model
  - a workhorse model in recent macro-finance literature: long-run risks model of Bansal and Yaron (2004) type
- ✓ Also need a wedge in order to generate adverse effects on the pricing kernel when the uncertainty rises above a certain threshold
  - the wedge factor: time-varying equity market liquidity following the tradition of Acharya and Pedersen (2005) and Brunnermeier and Pedersen (2008)

The Model

Epstein-Zin Recursive Utility

$$U_t = [(1-\beta)C_t^{\frac{1-\gamma}{\theta}} + \beta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}, \quad (1)$$

- β and γ are the time discount factor and the risk aversion parameter respectively
- $\theta = (1-\gamma)/(1-1/\psi)$ where ψ the intertemporal elasticity of substitution (IES)

Logarithm of the pricing kernel or IMRS

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \quad (2)$$

- $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ is the growth rate of aggregate consumption
- $r_{c,t+1}$ is the log of the return on an *imaginary* asset which delivers aggregate consumption as its dividend each time period.

The Model

Aggregate Consumption Process for Home

$$\begin{aligned} \Delta c_{t+1} &= \mu_g + x_t + \sigma_{g,t}\eta_{t+1} \\ x_{t+1} &= \rho x_t + \sigma_{x,t} e_{t+1} \\ \sigma^2_{g,t+1} &= v_g \sigma^2_{g,t} + \omega_{g,t+1} \\ \sigma^2_{x,t+1} &= v_x \sigma^2_{x,t} + \omega_{x,t+1}, \end{aligned} \tag{3}$$

- x_t is a persistent long-run expected growth component.
- e_t and η_{t+1}: long-run and short-run consumption shock respectively (i.i.d. standard normal)
- $\omega_{x,t}$ and $\omega_{g,t}$: long-run and short-run consumption volatility shock respectively (i.i.d. gamma)

Aggregate Dividend Process for Home

$$\Delta d_{t+1} = \mu_g + \phi x_t + \varphi_d \sigma_{g,t} \eta_{d,t+1}, \tag{4}$$

- the average dividend growth rate is equal to the rate for aggregate consumption, i.e., μ_g
- the volatility of dividend growth is simply φ_d times greater than the consumption counterpart.
- independence between η_{t+1} and $\eta_{d,t+1}$ is assumed.

The Model

Aggregate Equity Market Liquidity Process for Home

Definition of 'equity market liquidity' f_t: per-share cost of selling aggregate security, following Acharya and Pedersen (2005)

$$\begin{aligned} \Delta f_{t+1} &= -a x_t - \sigma_{\ell,t} \zeta_{t+1} \\ \sigma^2_{\ell,t+1} &= v_\ell \sigma^2_{\ell,t} + \omega_{\ell,t+1}. \end{aligned} \tag{5}$$

- ζ_{t+1} is the liquidity (level) shock
- $\omega_{\ell,t+1}$ is the liquidity (volatility) shock

Interpretation of Δf_{t+1}:

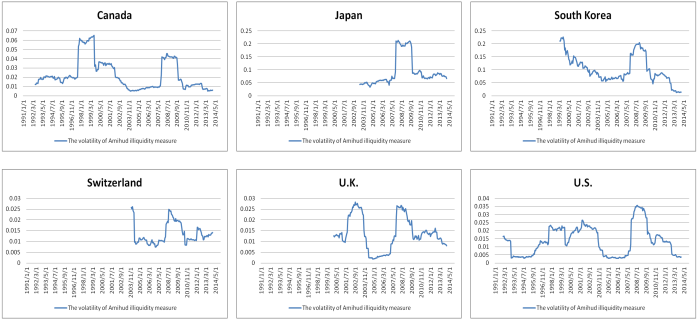
- $-a x_t$: pro-cyclical and persistent U.S. equity market liquidity provision; Brunnermeier and Pedersen (2008)
- ζ_{t+1}: shocks to aggregate transaction costs in equity markets; e.g., liquidity crunch episodes during Lehman Brothers in 2008, and the 'flash crash' in the U.S. stock market of 2010

Aggregate Equity Market Liquidity Process for Home

Interpretation of Δf_{t+1}:

- $\sigma_{\ell,t}$: time-varying nature of equity market transaction costs

Figure: Amihud illiquidity measures (volatility)



Liquidity-adjusted equilibrium equity prices: Acharya and Pedersen (2005)

✓ Acharya and Pedersen (2005) show that the equilibrium asset price process $\{P_t\}_{t=0}^{\infty}$ under exogenously given Δd_{t+1} and $\Delta f_{t+1}, \forall t$ is equivalent to the one under a imaginary dividend process of $\Delta d_{t+1} - \Delta f_{t+1}, \forall t$.

$$\begin{aligned} \Delta D_{t+1} &= \mu_g + (\phi + a)x_t + \varphi_d \sigma_{g,t} \eta_{d,t+1} + \sigma_{\ell,t} \zeta_{t+1}, \\ \Delta D^*_{t+1} &= \mu_g + (\phi + a)x_t + \varphi_d \sigma^*_{g,t} \eta^*_{d,t+1} + \sigma^*_{\ell,t} \zeta^*_{t+1} \end{aligned} \quad (6)$$

A variance-covariance matrix for $(\omega_{\ell,t}, \eta_t)$, Σ

$$\Sigma = \begin{bmatrix} \sigma^2_{\ell w} & \tau \\ \tau & 1 \end{bmatrix}$$

where $\tau < 0$. The same applies to the foreign counterpart.

Evidence for $\tau < 0$

Table: Correlations between SR economic growth and liquidity volatility

Countries	$corr(R, \sigma^2_{TED})$	$corr(R, \sigma^2_{lq})$
U.S.	-0.1592	-0.1905
Germany	-0.0432	-0.0648
U.K.	-0.0341	-0.2105
Switzerland	-0.1535	-0.0702
South Korea	0.0250	-0.0198
France	-0.0268	-0.3090
Austria	0.0007	-0.2040
Netherlands	-0.0832	-0.2694
Belgium	-0.0925	-0.2937
Japan	-0.0460	-0.2398
Canada	-0.1434	-0.1019

Note: σ^2_{TED} refers to the 2-year rolling variance of TED spreads.

Equilibrium Pricing Kernel or IMRS

Stochastic Discount Factor (SDF)

$$\begin{aligned} m_{t+1} = {} & m_0 + m_x x_t + m_{gs}\sigma_{g,t}^2 + m_{xs}\sigma_{x,t}^2 \\ & -\lambda_\eta \sigma_{g,t}\eta_{t+1} - \lambda_e \sigma_{x,t} e_{t+1} - \lambda_{gw}\omega_{g,t+1} - \lambda_{xw}\omega_{x,t+1}, \end{aligned} \quad (7)$$

- m_{gs}: the market price of current short-run volatility
- λ_η: the market price of short-run risks
- λ_{gw}: the market price of short-run **volatility** risks

$$m_{gs} = -\frac{1}{2}(\gamma - \frac{1}{\psi})(\gamma - 1) \begin{cases} = 0 & \text{if CRRA} \\ < 0 & \text{if } \gamma > 1 \text{ and } \psi > 1. \end{cases}$$

$$\lambda_\eta = \gamma,$$

$$\lambda_{gw} = -\left(\gamma - \frac{1}{\psi}\right)(\gamma - 1)\frac{\kappa_1}{2(1-\kappa_1 \nu_g)}, \begin{cases} = 0 & \text{if CRRA} \\ < 0 & \text{if } \gamma > 1 \text{ and } \psi > 1. \end{cases}$$

Equilibrium Real FX rate

Under complete FX market assumption;

$$s_{t+1} - s_t = m^*_{t+1} - m_{t+1}, \tag{8}$$

- s_t is the real FX rate in home currency per unit of foreign currency

Equilibrium Real FX Process

$$s_{t+1} = s_t + \underbrace{m_{gs}}_{<0} \left\{ \sigma^{*2}_{g,t} - \sigma^2_{g,t} \right\} - \underbrace{\lambda_\eta}_{>0} \left\{ \sigma^*_{g,t}\eta^*_{t+1} - \sigma_{g,t}\eta_{t+1} \right\} - \underbrace{\lambda_{gw}}_{<0} \left\{ \omega^*_{g,t+1} - \omega_{g,t+1} \right\}. \tag{9}$$

Equilibrium Real Equity Returns

$$r_{d,t+1} = \ell_0 + \ell_1 pd_{t+1} - pd_t + \Delta D_{t+1},$$
$$pd_t = B_0 + B_x x_t + B_{gs}\sigma^2_{g,t} + B_{xs}\sigma^2_{x,t} + B_{\ell s}\sigma^2_{\ell,t}, \tag{10}$$

- pd_t : the log price to (imaginary) dividend ratio
- B_{gs} and $B_{\ell s}$: the sensitivity of pd_t to current short-run consumption and liquidity volatility respectively

$$B_{gs} = \frac{0.5(\varphi_d - \gamma)^2 - (\gamma - 1/\psi)(\gamma - 1)}{1 - \ell_1 v_g} < 0, \quad \text{under } \varphi_d \approx \gamma > 1 \text{ and } \psi > 1 \tag{11}$$

$$B_{\ell s} = \frac{1}{2(1 - \ell_1 v_l)} > 0, \quad \text{similar to Pastor and Veronesi (2006)} \tag{12}$$

Equilibrium Risk Premium on Equities

Lemma

The risk premium has the following form.

$$RP_t = \underbrace{\lambda_e \ell_1 B_x \sigma_{x,t}^2}_{\textit{time-varying part}} + \lambda_{gw} \ell_1 B_{gs} \sigma_{gw}^2 + \lambda_{xw} \ell_1 B_{xs} \sigma_{xw}^2 + \underbrace{\lambda_\eta \ell_1 B_{\ell s} \tau}_{<0}$$

With IES and risk aversion both being larger than one, $\lambda_e \ell_1 B_x > 0$*,* $\lambda_{gw} \ell_1 B_{gs} > 0$*,* $\lambda_{xw} \ell_1 B_{xs} > 0$*, and* $\lambda_\eta \ell_1 B_{\ell s} \tau < 0$*.*

- $|\tau| \Uparrow \Rightarrow$ equity price and consumption process comove to a lesser extent $\Rightarrow$ less risk premium on equities

Correlations on FX and Equity Returns

Proposition

The conditional covariance of unexpected (or realized) FX movements and unexpected (or realized) relative equity returns have the following closed form solution in this model economy.

$$cov_t\left[FX_{t+1}, RD^*_{t+1}\right] = \underbrace{-\lambda_{gw} B_{gs}}_{<0} 2\left[\sigma^2_{gw} + (\bar{\omega}_g)^2\right] \underbrace{-\tau\lambda_\eta B_{ls}}_{>0}\left[\sigma^*_{g,t} + \sigma_{g,t}\right], \quad (13)$$

where $FX_{t+1} = s_{t+1} - s_t$ *and* $RD^*_{t+1} = r^*_{d,t+1} - r_{d,t+1}$.

Under the assumption that $\gamma > 1$, $\psi > 1$ *and* $\tau < 0$ *there exists a unique positive threshold level of* Q *such that if* $\sigma^*_{g,t} + \sigma_{g,t} > Q$ *then, the conditional covariance becomes a positive value, otherwise its sign is reversed. The* Q *is given by*

$$Q = \frac{-\lambda_{gw} B_{gs} 2\left[\sigma^2_{gw} + (\bar{\omega}_g)^2\right]}{\tau\lambda_\eta B_{ls}} > 0. \quad (14)$$

Correlations on FX and Equity Returns

Intuition:

$$cov_t\left[FX_{t+1}, RD^*_{t+1}\right] = E_t\left[(FX_{t+1} - E_t[FX_{t+1}])(RD^*_{t+1} - E_t[RD^*_{t+1}])\right] \tag{15}$$

$$FX_{t+1} - E_t[FX_{t+1}] = \underbrace{-\lambda_{gw}}_{>0}\left\{\omega^*_{g,t+1} - \omega_{g,t+1}\right\}\underbrace{-\lambda_{\eta}}_{<0}\left\{\sigma^*_{g,t}\eta^*_{t+1} - \sigma_{g,t}\eta_{t+1}\right\}, \tag{16}$$

$$\begin{aligned} RD^*_{t+1} - E_t[RD^*_{t+1}] = & \underbrace{B_{gs}}_{<0}\left\{\omega^*_{g,t+1} - \omega_{g,t+1}\right\}\underbrace{+B_{\ell s}}_{>0}\left\{\omega^*_{\ell,t+1} - \omega_{\ell,t+1}\right\} \\ & + \varphi_d\left\{\sigma^*_{g,t}\eta^*_{d,t+1} - \sigma_{g,t}\eta_{d,t+1}\right\} + \left\{\sigma^*_{\ell,t}\zeta^*_{t+1} - \sigma_{\ell,t}\zeta_{t+1}\right\}. \end{aligned} \tag{17}$$

Negative Correlation Factor: (realized) foreign CS volatility $\Uparrow \Rightarrow$ (realized) foreign SDF $\Uparrow$ and (realized) price/dividend ratio $\Downarrow \Rightarrow$ (realized) foreign currency appreciation and (realized) lower foreign equity returns

Positive Correlation Factor: (realized) foreign CS level $\Uparrow \Rightarrow$ (realized) foreign SDF $\Downarrow$ and (realized) foreign liquidity volatility $\Downarrow$ (though $\tau < 0$) $\Rightarrow$ (realized) foreign currency depreciation and (realized) lower foreign equity returns

Calibration of FX and Equity Returns: Parameterization

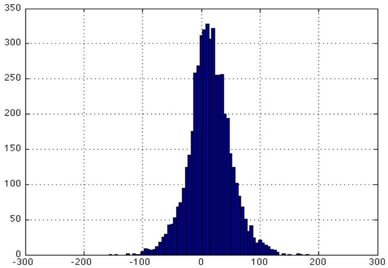
Consumption Dynamics	
Mean of consumption growth	$\mu_g = 0.0016$
Expected growth persistence	$\rho = 0.991$
Short-run volatility level	$\sigma_g = 0.0042$
Short-run volatility persistence	$v_g = 0.803$
Short-run volatility of volatility	$\sigma_{gw} = 1.57 * 10^{-5}$
Long-run volatility level	$\sigma_x = 1.67 * 10^{-4}$
Long-run volatility persistence	$v_x = 0.9799$
Long-run volatility of volatility	$\sigma_{xw} = 1.96 * 10^{-6}$
Aggregate Dividend and Liquidity Dynamics	
Aggregate dividend sensitivity to long-run news	$\phi + a = 1.25$
Aggregate dividend growth volatility level	$\varphi_d = 10$
Aggregate liquidity volatility level	$\sigma_\ell = 0.2$
Aggregate liquidity volatility persistence	$v_\ell = 0.97$
Covariance parameter for SR growth and liquidity volatility	$\tau = -0.0235$
Aggregate liquidity volatility of volatility	$\sigma_{\ell w} = 2.16 * 10^{-3}$
Preference Parameters	
Discount factor	$\beta = 0.9978$
Intertemporal elasticity of substitution	$\psi = 1.5$
Risk aversion coefficient	$\gamma = 10$

Calibration of FX and Equity Returns: Quantitative Results

✓ FX, equity returns, and the 'uncertainty index' simulation of a 20-year period

✓ Estimate the coefficient on the interaction term between equity return differentials and uncertainty index: γ as in the empirical section

Figure: Frequency distribution of regression coefficients, $\hat{\gamma}$, based on 5000 simulations



✓ Mean of 14 (about 10 in empirical evidence) and about 67% of positive $\hat{\gamma}$ (13 out of 19 pairs, 68% in empirical evidence)

Concluding Remarks

- ✓ I show that correlations between FX and equity returns are time-varying in data
- ✓ I propose one plausible theory to explain this evidence
- ✓ Key ingredients are: preferences for early resolution of uncertainty and negative correlations between short-run growth and equity market liquidity volatility
- ✓ Might be a useful framework to study the time-varying UIP conditional on market volatility
- ✓ Limitations and extensions:
  - Partial equilibrium approach on consumption volatility process
  - Equity market liquidity process is a kind of black box too
  - Nominal correlations also exhibit similar patterns: monetary factors?