**Was There an Explosive Bubble in U.S. Stock Prices before the Recent Stock Market Crash?**

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## Abstract

Existing studies on bubbles have been mainly concerned with investigating the stationarity properties of stock prices and dividends using unit-root and cointegration tests. However, the standard tests may not be able to detect an important class of bubbles. We develop a model that relates bubble measures to the Weibull distribution. In recent times there were at least three eruptions and subsequent collapses of seeming bubbles: 1987, 2000, and 2007. Using U.S. monthly data from 1980:1 to 2007:10, we have found that only the boom and crash of 2007 represented a bubble, although our stationarity tests fail to detect the bubble. Our results are in agreement with recent findings reported by Bohl (2003) and Nasseh and Strauss (2004).



Key Words: Intrinsic bubbles, extraneous bubbles, stationarity, unit roots, cointegration, Weibull distribution, bursting rate, hazard rate

JEL Classification Number: E44, G12

## 1. Introduction

The U.S. stock market has been increasingly volatile in recent years. During one infamous week in October, 2008 (October 6 – 10), the Dow Jones lost more than 18 percent of its value, and the Dow′s swing from high to low on October 10 was the biggest since the Dow was created in 1896. The recent crash of the U.S. stock market has followed the plateau of U.S. stock prices peaking in October 2007. There have been many episodes in history that may be adequately characterized as ′irrational exuberance′coined by former Federal Reserve chairman Alan Greenspan.

Bubbles can be divided into rational and irrational bubbles. Irrational bubbles mean that market participants do not recognize the existence of bubbles. Rational bubbles, on the other hand, occur when market participants recognize the existence of bubbles but expect increasing prices, which leads to continual increases in stock prices that are regardless of the internal value of firms. A rational bubble is a realization of self-fulfilling expectations of market participants. Even when the stock market seems to be overheated, investors are not inclined to conclude that bubbles will burst. People believe that stock prices will adjust into appropriate levels so far as they do not develop into explosive bubbles.

Earlier views on a bubble ascribe it to some psychological factors such as herd behavior, animal spirits or cognitive biases in which bubbles propagate themselves. A significant deviation of stock prices from the path predicted by market fundamentals may be due to waves of pessimistic or optimistic market psychology. Since this view attempts to link a bubble to some extraneous factors, this type of a bubble may be termed an extraneous bubble.

It has been widely believed that a bubble, especially an extraneous bubble does not lend easily itself to direct testing. Bubbles are recognized only when they burst. This nature of bubbles leads economists to acknowledge that it is difficult to test if there is a bubble or not. Evans (1991) maintains that it is difficult to test for the presence of a bubble when stock prices are too high but do not have the possibility of bursting. Flood and Hodrick (1990) have expressed a more pessimistic view:

> ″Whether the actual volatility of equity returns is due to time variation in the rational equity risk premium or to bubbles, fads, and market inefficiencies is an open issue. Bubble tests require a well-specified model of equilibrium expected returns that has yet to be developed, and this makes inference about bubbles quite tenuous.″

For reasons of this difficulty, economists traditionally used indirect methods to test for bubbles. For instance, Shiller (1981) used variance bounds tests and interpreted excessive deviations from the bounds as a bubble.

However, recent bubble models elaborated by Hamilton and Whiteman (1985), Diba and Grossman (1988a,b,c), Evans (1991), Froot and Obstfeld (1991) and others represent a significant departure from the conventional view in that they reinterpret rational bubbles in terms of market fundamentals. These bubbles may be termed intrinsic bubbles, as opposed to extraneous bubbles. One attractive feature of the intrinsic bubble specification may be found in its ability to obtain testable implications for bubbles.

A number of recent studies have investigated the existence of intrinsic bubbles in stock markets by examining the stationarity properties of stock prices and dividends on the basis of unit-root and cointegration tests. These studies include Campbell and Shiller (1987, 1988), Diba and Grossman (1988), Crowder and Wohar (1988), Froot and Obstfeld (1991), Craine (1993), Charemza and Deadman (1995), Timmermann (1995), Lamont (1998), Bohl (2003), Sarno and Taylor (2003), Nasseh and Strauss (2004), Koustas and Serletis (2005), Cunado, Gil-Alana, and de Gracia (2005, 2007), among others.

However, the existing bubble models still remain unsatisfactory. The empirical relevance of these intrinsic bubble models has been seriously undermined by the results of Evans (1991) who has criticized that unit-root approaches are unable to detect an important class of rational bubbles. Evans has shown that even in the presence of bubbles, stock prices could be integrated and cointegrated with market fundamentals, so that standard unit-root and cointegration tests are not able to detect this class of rational bubbles. This view has been echoed by Charemza and Deadman (1995) and Ackert and Smith (1993). Ackert and Smith have noted that conventional measures of dividend payments to shareholders grossly underestimate the total cash flow to shareholders, and this underestimation of cash flows could have impacted conventional types of cointegration between dividends and prices, which may bias the tests to show no cointegration.

More interestingly, Rappoport and White (1991) have pursued a different strategy for testing for intrinsic bubbles. Their approach is notably distinguished from standard tests by directly extracting an estimate of the path of the bubble and its probability of bursting. Abreu and Brunnermeier (2003) also provide a setting in which new events can have a disproportionate impact relative to their intrinsic informational content without reference to the stationarity processes of stock prices and market fundamentals.

The main purpose of this study is to develop a simple method of testing for rational bubbles that does not rely on cointegrating relations among variables in asset pricing models. To this end, we directly derive a bubble measure from the information error model and test whether the bubble measure is explosive. Our approach parallels that of Rappoport and White in that we extract a bubble measure and relate it to the probability of bursting in the context of the Weibull distribution and that of Abreu and Brunnermeier in that bubbles can arise as a result of informational errors.

We have tested for the existence of explosive bubbles in the U.S. stock market using monthly data from January 1980 to October 2007. Our empirical analysis for three sub-samples (1980:1 – 1987:10, 1987:11 – 2000:8, and 2000:9 – 2007:10) shows that only the surge of stock prices peaking in October 2007 contained an explosive bubble.

## 2. A Review of the Literature

Shiller (1981), using the present value (PV) model of stock prices, has demonstrated that stock prices are too volatile to be consistent with the present value of rationally expected future dividends discounted by a constant real interest rate. In particular, Shiller's variance bounds tests establish that the variance of the market price of a stock should not be greater than that of the present discounted value of future cash flows. Shiller has shown that the violation of this inequality is overwhelming for the U.S. data. The violation of the variance bounds is interpreted as rejection of the efficient markets hypothesis.

Hamilton and Whiteman (1985) have dealt with bubbles that can be dependent on market fundamental values for the first time. They have discussed bubbles from the perspective of the stationarity properties of stock prices and market fundamentals. Their argument is that it is not possible to test the proposition that bubbles are caused by self-fulfilling expectations from purely extraneous factors. The existence of bubbles is the output of rational reactions to market fundamentals by market participants. Their proposition implies that there exists a bubble if d-th differenced fundamental values are stationary, but d-th differenced stock prices are nonstationary. Put it differently, there exists a bubble if stock prices are more explosive than fundamental values such as dividends.

Diba and Grossman (1988) postulate that if there is no bubble premium in stock prices, then stock prices should be cointegrated with market fundamentals in a nonlinear fashion (nonlinear cointegration). However, they also suggest that the opposite is not true since non-cointegrated stock prices and dividends can occur from the nonstationarity of the unobservable variable which can be observed by market participants, but cannot be observed by researchers. Diba and Grossman have conducted cointegration tests for the U.S. data of the S&P 500 composite price index for the period 1871-1986 and found that the U.S. stock prices do not contain explosive rational bubbles.

Campbell and Shiller (1987) have tested for cointegration between stock prices and dividends using annual data for the S&P 500 index from 1871 to 1986, and found persistent deviations of stock prices from the present value model, which can be interpreted as evidence for the presence of rational bubbles in U.S. stock prices. In their subsequent paper, Campbell and Shiller (1989) have suggested that the dividend-price ratio (D/P) can be explained by some market fundamentals. They have tested for the model using S&P data for 1871-1986 and 1926-1986 and found that the log dividend-price ratio has a significant relationship with the growth of dividends. Their results further indicate that there is also substantial unexplained variation in the log dividend-price ratio.

Evans (1991) has conducted the simulation of two hundred replications, each of 100 'years' of stock price and dividend data and shown that the cointegration of stock prices with dividends cannot be viewed as evidence against the presence of bubbles. His Dickey-Fuller unit-root tests indicate that real dividends (d) have a unit root and Δd is stationary. For stock prices (P), the results are similar. Furthermore, P and d appear to be clearly cointegrated. This test procedure thus fails to find the presence of the bubble, since the existence of rational bubbles means that price series are 'less stationary' or 'more explosive' than dividend series. However, simulations show that over the sample, there appear to be four bubble eruptions, each followed by a collapse.

Froot and Obstfeld (1991) have maintained in line with Hamilton and Whiteman that since a bubble is the output of market participants′ rational reaction to market fundamentals, bubbles are not extraneous. On the contrary, any self-fulfilling expectations of intrinsic bubbles are based on market fundamental values. They have tested for a unit root in the price-dividend ratio using the S&P 500 index for the period 1900-1988. Using the Phillips-Perron (1988) unit-root tests, they were unable to reject the unit root null hypothesis in five of six cases, which indicates the lack of cointegration betweens stock prices and dividends. Thus, they have concluded that deviations from the PV model are permanent—consistent with the presence of rational bubbles in stock prices.

Craine (1993) has argued that a unit root in the price-dividend ratio (P/D) violates the no rational bubbles restrictions and applied the Augmented Dickey-Fuller (1981) unit-root test to the price-dividend ratio using annual S&P 500 data from 1876 to 1988. His test results show that either the price-dividend ratio contains a rational bubble or the discount factor must be stochastic and contain a large predictable component.

Yuhn (1997) has argued that Campbell and Shiller′s (1987) linear cointegrating relation between stock prices and dividends is not appropriate for investigating stock price volatility and derived a dynamic form of cointegration between stock prices and dividends. His empirical results reveal that little evidence for linear cointegration is found, but the evidence of nonlinear cointegration is overwhelming for U.S. monthly data from 1959:1 to 1992:6, indicating no volatility in the U.S. stock market during the sample period.

More recently, Sarno and Taylor (2003) have examined the existence of rational bubbles in Latin American emerging markets—Brazil, Chile, Columbia, Mexico, and Venezuela. They fail to reject the null hypothesis of no cointegration—i.e., the hypothesis of the presence of bubbles at the 5% level for any of countries, providing strong evidence for the existence of bubbles in each of the Latin American stock markets.

Bohl (2003) has studied the presence of a bubble in annual (1871-1999) and monthly (1871-2001) U.S. stock prices using the momentum threshold autoregressive (MTAR) model developed by Enders and Granger (1998) and Enders and Siklos (2001). His empirical findings indicate the absence of periodically collapsing bubbles in the U.S. stock market over the 1871-1995 period. However, the evidence for the sample including the rapid share price increases since the middle of the 1990s (1871-2001) is interpretable in favor of the existence of periodically collapsing bubbles in U.S. stock prices.

Nasseh and Strauss (2004) have applied panel cointegration testing and estimation methods to quarterly data for 84 firms over the 1979-1999 period to examine the long-run relation between stock prices and dividends. Their results show that there is an approximately one-for-one long run relation (cointegrating relation) between stock prices and dividends for large established firms. However, their test results show that stock prices are overvalued by 43% during the late 1990s. They have found that real interest rates do not explain recent stock price overvaluations. Instead, declines in nominal interest rates explain approximately half of the overvaluations and a break in dividend payments in the mid-1990s can explain the remaining stock price overvaluation.

Koustas and Serletis (2005) have examined the behavior of the dividend-price ratio (D/P) or the dividend yield to test for the existence of bubbles. They have maintained that the log dividend yield ($d_t - p_t$) is a stationary stochastic process under the no rational bubble restriction. On the contrary, the presence of a unit root in the log dividend yield is consistent with rational bubbles in stock prices. They have applied fractional integration techniques to the S&P 500 log dividend yield for the period 1871 to 2000. Their findings yield robust rejections of the null hypothesis of rational bubbles, strongly suggesting that the log dividend yield is mean reverting.

Cunado et.al. (2005) have investigated whether the NASDAQ composite index and its corresponding dividend yield satisfy the condition required for the absence of rational bubbles. They have used the dividend-price ratio (D/P) model and applied fractional integration methods to monthly, weekly, and daily data over the period 1994:06-2003:11. When monthly data are used, the unit root null hypothesis cannot be rejected. However, when they use daily and weekly data, the order of integration appears to be higher than 0 but smaller than 1, suggesting that a certain degree of fractional cointegration exists between the two variables. This finding leads them to reject the existence of rational bubbles.

Cunado et. al. (2007) have tested for the existence of bubbles in the S&P 500 index for the period 1871:1-2004:6. They have used fractional integration techniques, allowing for structural breaks and a nonlinear adjustment process of prices to dividends. They have found orders of integration for the log price-dividend ratio to be equal to or higher than 1 and concluded that there exists a stock market bubble in the S&P 500 index over the entire period.

Unlike the existing studies that are primarily concerned with the stationarity properties of stock prices and dividends, Rappoport and White (1991) and Abreu and Brunnermeier (2003) depart from the standard tests by focusing on the nature of bubble measures. Rappoport and White have argued that although standard tests find no bubbles in the stock price data for the last 100 years, historical accounts, focusing on briefer periods, point to the existence of a bubble during the Great Depression period of 1928-29. Their approach has used the behavior of the premia demanded on loans collateralized by the purchase of stocks as a bubble measure.

Abreu and Brunnermeier (2003) have developed a dynamic game model in which bubbles can persist even though all rational arbitrageurs know that the stock price is too high, and they jointly have the ability to correct the mispricing. There can be a large and long-lasting departure from fundamental values, because there is dispersion of opinions among rational arbitrageurs concerning the timing of the bubble. Their model provides a setting in which ′overreaction′ and self-feeding price processes lead to full-fledged crashes.

Our model shares some common theme with Rappoport and White (1991) and Abreu and Brunnermeier (2003) in that our model extracts a bubble measure as market participants′ overreaction to the new information on market fundamentals.

## 3. The Theoretical Model

The present value (PV) model of stock prices implies that stock prices are equal to the present value of future cash flows such as dividends discounted by a constant real interest rate. Generally, the present value model takes the form of

(1) $$P_t = \delta E_t(P_{t+1} + D_{t+1})$$

where $P_t$ represents real stock prices at time *t* and $D_{t+1}$ indicates real dividends between *t* and *t*+1, and $\delta$ is a discount factor. The discount factor is equal to 1/(1 + *r*) with a constant real interest rate of *r*. The solution to equation (1) is given by

(2) $$P_t = \sum_{k=1}^{\infty} \delta^k E_t(D_{t+k}) + \lim_{T\to\infty} E_t \delta^T P_{t+T}$$

If we impose a transversality condition on equation (2), then we obtain the unique solution to equation (1), which is given by

(3) $$P_t = \sum_{k=1}^{\infty} \delta^k E_t(D_{t+k})$$

If the transversality condition fails to hold, we have a bubble part:

(4) $$P_t = \sum_{k=1}^{\infty} \delta^k E_t(D_{t+k}) + B_t$$

where $B_t = \lim_{T\to\infty} E_t \delta^T P_{t+T}$ measures the bubble term and satisfies the following process:

(5) $$B_t = \delta E_t B_{t+1}$$

However, Hamilton (1986) and Diba and Grossman (1988a) reformulate the PV model in a different way from the conventional one such as equation (4). Hamilton (1986) has proposed the following PV model:

(6) $$P_t = \delta(D_{t+1} + E_t P_{t+1} + \pi_t)$$

where $\pi_t$ can be viewed as a catch-all random variable that is not observed by researchers, but can be observed only by market participants. The random variable includes the real interest rate, the risk premium, taxes on dividends, etc. $\pi_t$ is assumed to be stationary, that is, I(0).

Diba and Grossman (1988a) have conducted an empirical investigation of the Hamilton model using a slightly modified version.

(7) $$P_t = \delta E_t(P_{t+1} + \alpha D_{t+1} + \pi_{t+1})$$

Diba and Grossman have proposed that if the unobserved $\pi_t$ is stationary, and first differenced dividends and first-differenced stock prices are stationary, there does not exist a bubble. That is, there is no bubble if $\pi_t$ is I(0), and stock prices and dividends are I(1), and stock prices and dividends are cointegrated (CI(1,1)). Diba and Grossman have derived the following estimation equation:

(8) $$P_{t+1} + \gamma D_{t+1} - (1/\delta)P_t = e_{t+1} - \pi_{t+1}$$

where $e_{t+1} = P_{t+1} + \nu D_{t+1} + \pi_{t+1} - E_t(P_{t+1} + \nu D_{t+1} + \pi_{t+1})$ .The left-hand side becomes stationary if $\pi_{t+1}$ is stationary, since $e_{t+1}$ is not serially correlated.

In this study, we develop a rational bubble model in the spirit of Hamilton (1986) and Diba and Grossman (1988) but in a more straightforward manner. Our bubble model can be viewed as a significant improvement over the Hamilton and Diba-Grossman approaches. The obvious merit of our approach is that we derive the bubble measure without introducing unobservable random variables. Our model formulates a bubble measure as an overreaction to new information on market fundamentals by rational participants.

Since the present value relation must hold in period t - 1, we have

(9) $$P_{t-1} = \sum_{k=1}^{\infty} \delta^k E_{t-1}(D_{t+k-1})$$

Multiplying equation (3) by $\delta$ and subtracting it from equation (9),

(10) $$P_{t-1} - \delta P_t = E_{t-1}\delta D_t - \sum_{k=1}^{\infty} \delta^{k+1}[E_t - E_{t-1}]D_{t+k}$$

We can rearrange equation (10) to obtain

(11a) $$\delta(P_t + D_t) - P_{t-1} = \sum_{k=0}^{\infty} \delta^{k+1}[E_t - E_{t-1}]D_{t+k}$$

(11b) $$P_t + D_t = \frac{1}{\delta}P_{t-1} + \sum_{k=0}^{\infty} \delta^k [E_t - E_{t-1}]D_{t+k}$$

Equation (11b) can be further simplified as

(12) $$P_t = (1+r)P_{t-1} - D_t + \nabla_t$$

where $\nabla_t =$ denotes the present value of the sum of the forecast errors of market participants. That is, this is the rational response of participants to market fundamentals

since is the difference between the k-period-ahead forecast of dividends in the previous period and the k-period-ahead forecast of dividends based on the arrival of new information in the current period. Equation (12) will be called an information error model.

Thus, the term, $\nabla_t$, is used as a measure of bubbles since an intrinsic bubble is an overreaction of participants to new information on market fundamentals.

It can be demonstrated that $\nabla_t$ is the same as the unobserved variable in the Hamilton-Whiteman and Diba-Grossman models. We can rewrite our information error model as follows:

(13) $$P_{t+1} = \frac{1}{\delta} P_t - E_t D_{t+1} + v_{t+1}$$

where $v_{t+1} = \sum_{k=1}^{\infty} \delta^k [E_t - E_{t-1}] D_{t+k+1}$. Let $E_{t+1} D_{t+1} - E_t D_{t+1} = e_{t+1}$

(14) $$P_{t+1} + D_{t+1} - \frac{1}{\delta} P_t = e_{t+1} + v_{t+1}$$

This expression is the same as the Diba-Grossman model when $v$ = 1 in their model. We note that $\nabla_{t+1} = e_{t+1} + v_{t+1}$ ($\nabla_{t+1} = e_{t+1} - \pi_{t+1}$ in the Diba-Grossman model). Thus, we can extract bubbles from $P_t + D_t - \frac{1}{\delta} P_{t-1}$ without introducing arbitrarily an unobserved random variable. Furthermore, $\nabla_t$ is not serially uncorrelated. This can be proved by the law of iterated expectations.

It is assumed that the bubble measure ($\nabla_t$) has a Weibull distribution. There is not only a parallel between the burst of a speculative bubble and a material's burning out, but also there is a good reason to believe that the bubble measure can be appropriately modeled as the Weibull function. A bubble is a rare event. Like other rare events, bubbles can be formulated in terms of the instantaneous rate at which an event occurs after duration t since some prior event has occurred.

We will denote a bursting bubble by a continuous random variable *T*. If *T* has a probability density function *f*(*t*), then the probability that optimistic expectations about stock prices continue to hold until a specific time *t* is given by

(15) $$z(t) = \Pr(T > t) = \int_t^{\infty} f(t)dt = 1 - F(t)$$

where $z(t)$ is the survival function for optimistic forecasting until a specific time $t$, and $F(t)$ is the cumulative density function. Then the following rate called the hazard rate measures the likelihood of the bursting of a bubble in the next small unit of time $\Delta t$, given that a bubble has survived until time t.

(16) $$\theta(t) = \lim_{\Delta t \to 0} \frac{[F(t+\Delta t) - F(t)]}{\Delta t} \cdot \frac{1}{z(t)} = \frac{F'(t)}{z(t)} = \frac{f(t)}{z(t)} = \frac{f(t)}{1 - F(t)}$$

Thus, the busting rate is given by

(17) $$\theta(t) = -\frac{z'(t)}{z(t)} = -\frac{d \ln z(t)}{dt}$$

From equation (17), we can derive the probability density function (*pdf*) as follows:

(18a) $$\ln z(t) = -\int \theta(t)dt + \ln c$$

Equivalently,

(18b) $$z(t) = ce^{-\int \theta(t)dt}$$

Then the cumulative density function can be expressed as

(19) $$F(t) = 1 - ce^{-\int \theta(t)dt}$$

If $f(t)$ follows the Weibull distribution, then we have the following *pdf*

(20) $$f(t) = \alpha \lambda t^{\alpha-1} \exp(-\lambda t^{\alpha})$$

and the bursting rate is given by

(21) $$\theta(t) = \alpha \lambda t^{\alpha-1}$$

When λ is equal to 1, we have $\theta(t) = \alpha t^{\alpha-1}$, which gives the possibility of an extraneous bubble (speculative bubble). $\theta(t)$ can be greater than one since $\theta(t) \cdot \Delta t$ is equal to the conditional probability. The coefficient $\alpha$ denotes the shape parameter, also known as the Weibull slope. Different values of the shape parameter can have significant effects on the behavior of the distribution. In fact, different values of the shape parameter may lead to different distributions. For example, when $\alpha = 1$, the *pdf* of the two-parameter Weibull reduces to that of the one-parameter exponential distribution.

The bursting rate $\theta(t)$ will increase or decrease depending on the value $\alpha$. There are four possibilities.

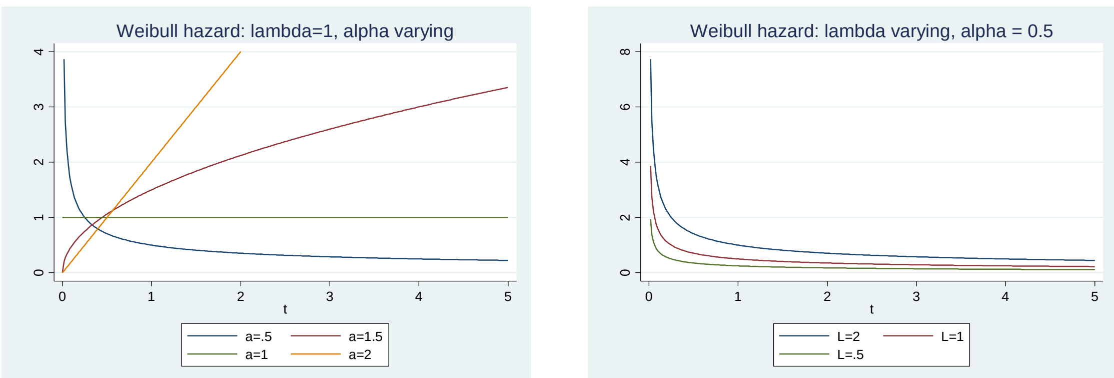
(a) If $\alpha$ is smaller than one, the bursting rate decreases, so that the possibility of bursting will also decrease as time goes on.

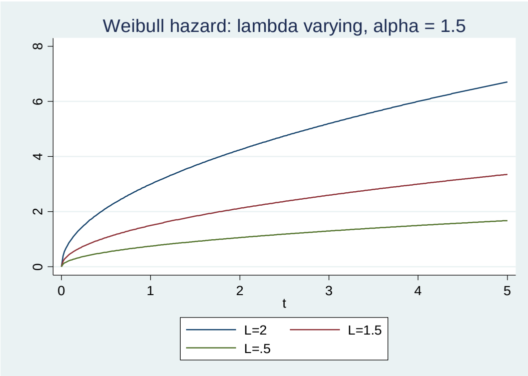
(b) If $\alpha$ is equal to one, the bursting rate is constant, so that the possibility of bursting will also be constant. This case is the same as the well-known exponential distribution.

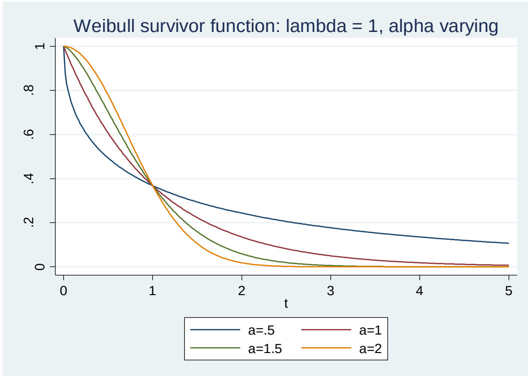
(c) If $1 < \alpha < 2$, the bursting rate increases at a decreasing rate, so that the possibility of bursting also will increase at a decreasing rate.

(d) If $\alpha$ is greater than 2, the bursting rate increases at an increasing rate, so that the possibility of bursting will increase at an increasing rate. Thus, an explosive bubble occurs when $\alpha$ is greater than 2. <Figure 1> shows the Weibull distribution for various values of the parameters.

<Figure 1> Weibull Distributions






We can derive the specifications for purely extraneous bubbles and intrinsic bubbles from equation (21). We obtain an intrinsic bubble model if we let $\lambda = \exp(X'\beta)$ and an extraneous bubble model if we let $\lambda$ = 1:

(22a) $$\theta(t) = \alpha t^{\alpha-1}\exp(X'\beta) = \psi_1(t)\psi_2(X'):\quad \text{Intrinsic bubbles}$$

(22b) $$\theta(t) = \alpha t^{\alpha-1} = \psi_1(t) \text{ with } \psi_2(X') = 1:\quad \text{Extraneous bubbles}$$

where $X$ denotes the set of variables that are thought to be related to a firm's fundamental values.

## 4. Empirical Analysis

### 4.1. Data Description

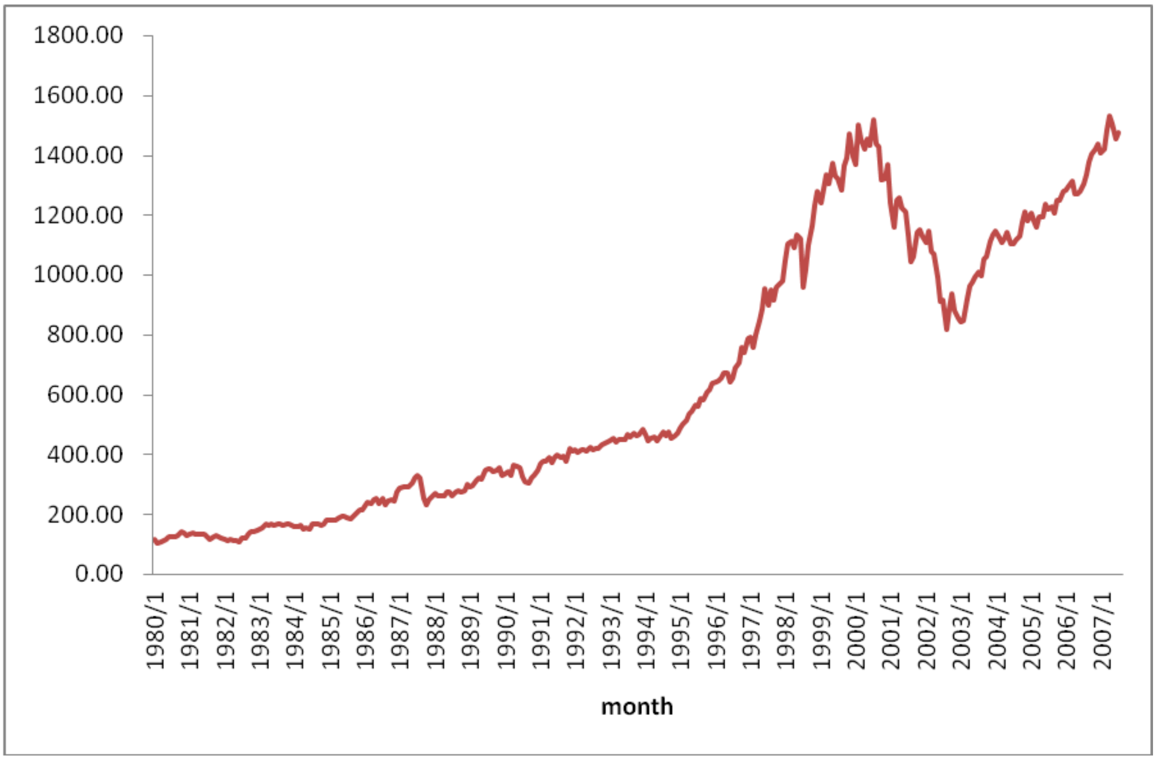
We have used U.S. monthly data from 1980:1 to 2007:10. We have divided the sample into three sub-samples: 1980:1 - 1987:10 (period 1), 1987:11 - 2000:8 (period 2), and 2000:9 - 2007:10 (period 3). This breakdown of the sample coincides with the eruptions and subsequent collapses of seeming bubbles in the U.S. stock market in modern times. The stock price series reached a peak in October 1987 followed by a downturn commonly known as the Black Monday, and it reached another peak in August 2000 followed by the collapse of the stock prices together with the burst of the IT bubble. The third peak point on the series during the sample period occurred in October 2007.

The data used in this study include the S&P 500 composite index, the price-earnings ratio

(PER), the term spread (TERM) between the short-term interest rate (3-month T-bill rate) and the long-term interest rate (10-year T-bond rate), the default rate (DEF), the nominal effective exchange rate (EX), and the unemployment rate (UNEMP). The data have been obtained from the Datastream database and the International Financial Statistics. The price-earnings ratio (PER) is derived by dividing the total market value of an index by the total amount of earnings. The default rate (DEF) is the default rate on all U.S. corporate bonds.

The term spread measures the forecast of the future economy. In a well-developed market, short-term interest rates reflect policy interest rates, but long-term interest rates include market participants′ expectations of the future state of the economy. Thus, the term premium is an index of market participants' forecasting of the economy. The term spread contains information on future inflation and economic growth. The following diagram shows movements in the S&P 500 index from January 1980 to December 2007.

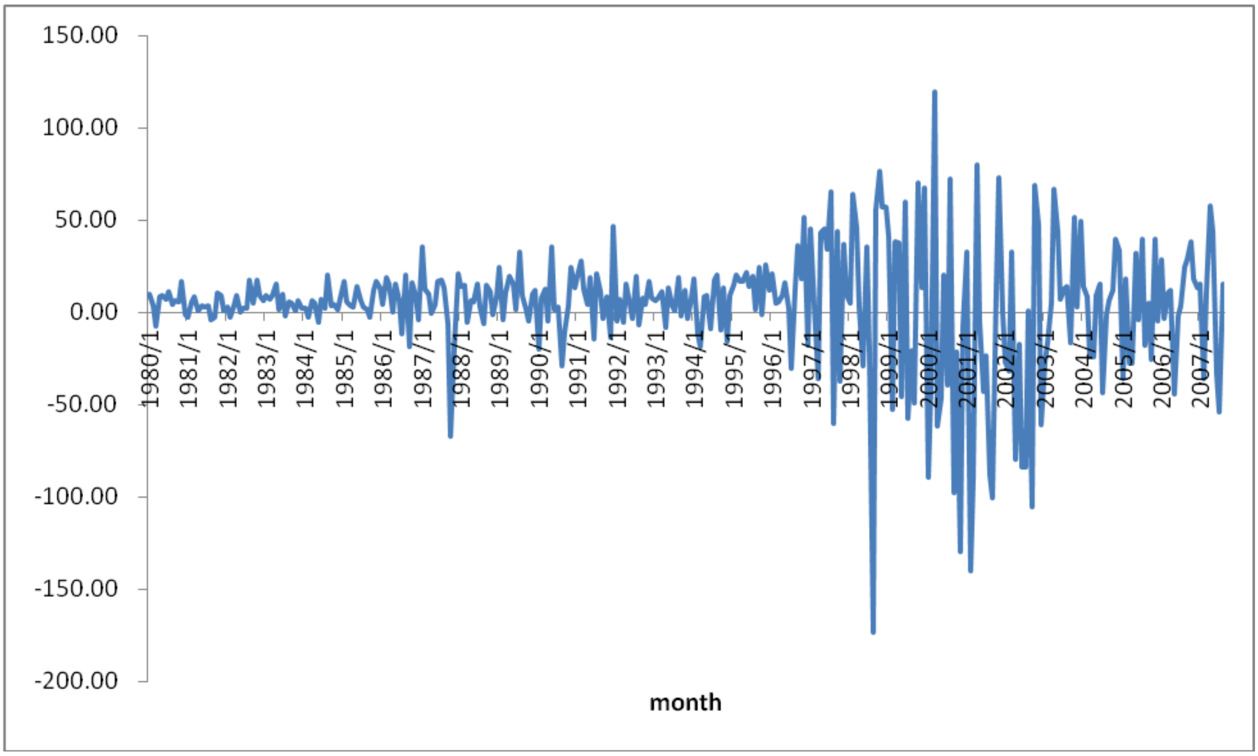
<Figure 2> S&P500 Index




### 4.2. Measuring Bubbles

We obtain the portion of unexplained variations of stock prices (bubble measures) from the information error model. It is not possible to judge whether stock prices are 'too high' or 'too low' without any criteria. The information error model gives a criterion that tells us whether current stock prices are too high or too low compared with the path implied by market fundamentals. Our information error model enables us to divide current stock prices into two parts, the market fundamental measure that is given by $P_t = -D_t + \frac{1}{\delta} P_{t-1}$ and the bubble measure that deviates from the expected orbit of the information error model. The decomposition of stock prices into these two parts is available from the authors upon request.

<Figure 3> Non-Fundamentals in Stock Prices




### 4.3. Empirical Results

#### A. Unit-Root Tests

It is important to check whether a time series under consideration is stationary. If a variable is stationary, it does not contain a unit root. In this paper, we use the Phillips-Perron test (PP) that allows for serial dependence and heterskedasticity. We have conducted unit-root tests for the following data-generating processes.

(23a) $$y_t = \mu + \alpha y^*_{t-1} + u^*_t$$

(23b) $$y_t = \mu + \beta(t - T/2) + \alpha y_{t-1} + u_t$$

<Table 1> Unit-Root Tests

| Variables | Phillips-Perron Test |
|---|---|

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1980:1-2007:10 | 1980.1-1987.10 | 1987.11-2000.8 | 2000.9-2007.10 | 1980:1-2007:10 | 1980.1-1987.10 | 1987.11-2000.8 | 2000.9-2007.10 |
| Bubble Measure | -18.22** | -6.30** | -15.20** | -7.86** | -18.29** | -6.33** | -15.62** | -8.69** |
| PER | -2.15 | -1.11 | -0.93 | -1.53 | -2.33 | -2.25 | -1.87 | -2.34 |
| Term Premium | -4.59** | -3.47** | -2.41 | -1.43 | -4.54** | -3.78** | -2.39 | -2.20 |
| Unemployment Rates | -1.58 | -1.36 | -0.32 | -2.00 | -2.81 | -2.05 | -1.17 | -2.52 |
| Default Rates | -2.10 | -1.34 | -1.46 | -0.24 | -1.85 | -2.47 | -1.39 | -2.62 |
| Exchange Rates | -2.35 | -1.51 | -1.14 | -0.34 | 0.94 | -0.42 | -2.88 | -3.02 |

*indicates significance at the 1% level and ** significance at the 5% level.

The tests have produced actually the same results for (23a) and (23b).

(1) The bubble measure is unambiguously stationary. The bubble measure has turned out to be stationary in both the data-generating processes (the process with only a constant and the process with both a constant and a trend) in every sub-period as well as in the entire sample period.

(2) The term spread is stationary only in period 1 and in the entire period.

(3) We fail to reject the unit root null hypothesis for all other variables.

(4) Thus, all other variables except for the bubble measure in all sample periods and the term spread (in period 1 and the entire period) contain a unit root. It is necessary to make these nonstationary variables stationary before we run a regression. These nonstationary variables have been first-differenced. We have confirmed that these first-differenced variables are stationary.

**B. Tests for Bubbles: The Weibull Tests**

A bubble of stock prices can be detected only when there is the possibility of bursting of the bubble. As long as there is no possibility of explosion although stock prices are very high, such high stock prices will adjust to an appropriate level predicted by market fundamentals with the passage of time. As we have seen from the hazard model, there is the possibility of bursting if $\alpha$ is greater than 2.

**B.1. Testing for Purely Extraneous Bubbles**

<Table 2> Tests for Extraneous Bubbles

| Period | 1980:1-2007:10 | 1980:1-1987:10 | 1987.11-2000:8 | 2000:9-2007:10 |
|---|---|---|---|---|
| $\alpha$ | $1.32^{*}$ | $1.37^{*}$ | $1.48^{*}$ | $1.64^{*}$ |

* indicates significance at the 5 percent level.

Purely extraneous bubbles are from participants' psychological wave of moods, not from the current or future state of the economy. It is clear that if $\alpha$ is greater than 2 in equation (22b) and significant, the explosion of a bubble can be caused by extraneous factors that are not related to market fundamentals. <Table 2> shows that there is no evidence for extraneous bubbles in the U.S. stock market, indicating that there is no possibility of seemingly too high stock prices to become explosive bubbles driven by a wave of people′s psychological biases or fads. The estimated value of $\alpha$ is 1.32 for the entire sample period, and 1.37 for sample period 1, 1.48 for sample period 2, and 1.64 for sample period 3. All the estimates are significant at the 5 percent level of significance. Thus, we can conclude that although stock prices seem to have been overvalued on certain time intervals, overheated stock prices were not driven by extraneous factors.

**B. 2. Testing for Intrinsic bubbles**

In order to investigate whether bubbles are driven by market fundamentals or not, we have included a set of variables in the hazard model that affect stock prices. These market fundamental variables include the PER (price-earnings ratio), TERM (term spread), UNEMP (unemployment rate), DEF (default rate) and EX (exchange rate). We have estimated the following hazard model with nonstationary variables differenced:

(24) $$\theta(t) = \alpha t^{\alpha-1} \exp\left(\beta_0 + \beta_1 PER_t + \beta_2 TERM_t + \beta_3 UNEMP_t + \beta_4 DEF_t + \beta_5 EX_t\right)$$

<Table 3> Tests for Intrinsic Bubbles

| | With the results of the unit root test | | | |
|---|---|---|---|---|
| Period | 1980:1-2007:10 | 1980:1-1987:10 | 1987:11-2000:8 | 2000:9-2007:10 |
| α | 1.52* | 1.78* | 1.61* | 1.92* |
| $\beta_0$ | 0.80 | -0.39 | 1.09 | -9.14* |
| PER | 0.06* | -0.05 | 0.01 | -0.13* |
| TERM | 0.04 | -0.05 | 0.34* | 0.13 |
| UNEMP | -0.36* | -0.39* | -0.49* | 0.21 |
| DEF | 0.11* | -0.18 | -0.01 | 0.34 |
| EX | -0.02* | 0.06* | -0.01 | 0.08 |

* indicates significance at the 5 percent level.

The key element in the Weibull distribution is the size of α. When the value of α is greater than 2, then the bursting rate increases at an increasing rate, and this is interpretable in favor of the existence of an explosive bubble. We have found the following results:

(1) The value of α is 1.52 for the entire sample period, and the coefficient is significant. Thus, we can argue that no evidence in favor of explosive bubbles is found for the entire sample period.

(2) The value of α is considerably less than 2 at the 5 percent level of significance in both the sample period 1 (1980:1-1987:10) and sample period 2 (1987:11-2000:8). Thus, during these briefer sub-sample periods, stock prices seem to have risen too much, but they did not develop into explosive bubbles. The spike of stock prices followed by the largest single-day drop during the 1987 Black Monday was a temporary adjustment of the market. The boom of the stock market starting in the middle of the 1990s and lasting until August 1980 reflected productivity growth brought by the IT (information technology)

Revolution and did not represent an explosive bubble.

(3) Interestingly enough, the value of α is 1.92 for the sample period 3 (2000:9-2007:10) and significant at the conventional level of significance. Since the estimate is so close to 2 that this finding can be construed as evidence for the existence of an explosive bubble during the 2000:9-2007:10 period. This finding is consistent with some recent studies such as Bohl (2003) and Nasseh and Strauss (2004) who have found that explosive bubbles were present in the U.S, stock market when the sample data include recent stock price hikes since the late 1990s. As observed by Rappoport and White (1991), it is entirely possible to detect a bubble in briefer sub-sample periods even though bubbles may not be present over a longer period of time that encompasses the shorter sub-sample periods.

(4) We have tested for the model using a broader pool of market fundamentals including industrial production and earnings per share, but this extension has little effect on the test results.

**C. Tests for Bubble: Unit-Root and Cointegration Tests**

<Table 4> Phillips-Perron Unit-Root Test for Prices (P) and Dividends (D)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Period | 1980:1-<br>2007:10 | 1980:1-<br>1987:10 | 1987:11<br>-2000:8 | 2000:9-<br>2007:10 | 1980:1-<br>2007:10 | 1980:1-<br>1987:10 | 1987:11<br>-2000:8 | 2000:9-<br>2007:10 |
| P | 0.129<br>(0.974) | -0.753<br>(0.827) | 3.152<br>(1.000) | -1.081<br>(0.720) | -1.912<br>(0.646) | -2.262<br>(0.450) | -0.184<br>(0.993) | -2.928<br>(0.159) |
| D | 3.395<br>(1.000) | 1.179<br>(0.998) | -2.117<br>(0.238) | 1.256<br>(0.998) | 1.386<br>(1.000) | -3.077<br>(0.118) | -6.013<br>(0.000) | -2.205<br>(0.325) |
| ΔP | -18.716<br>(0.000) | -6.584<br>(0.000) | -14.319<br>(0.000) | -8.334<br>(0.000) | -18.732<br>(0.000) | -6.329<br>(0.000) | -16.819<br>(0.000) | -8.798<br>(0.000) |
| ΔD | -25.865<br>(0.000) | -18.682<br>(0.000) | -24.066<br>(0.000) | -12.218<br>(0.000) | -28.433<br>(0.000) | -13.910<br>(0.000) | -25.953<br>(0.000) | -14.604<br>(0.000) |

*Numbers in parentheses denote p-values.

Following the traditional approach to testing for the existence of explosive bubbles, we have also conducted unit-root and cointegration tests for stock prices and dividends (D). We have examined two data-generating processes: processes with only a constant and processes with both a constant and a trend. Our Phillips-Perron unit-root tests reveal that stock prices unambiguously contain a unit root in both the processes for all sample periods, and the results for dividends are very similar to those of stock prices only with the exception that the dividend series (with both a constant and a trend) is stationary during sample period 2. On the other hand, first-differenced stock prices and first-differenced dividends are stationary in all data-generating processes for all samples.

We have tested for cointegration between stock prices and dividends using the Augmented Engle-Granger test. We are unable to reject the unit-root null hypothesis for the entire sample period as well as for sub-sample periods 1 and 2, but we reject the unit-root null hypothesis for sub-sample period 3. According to the unit-root and cointegration test results, we should conclude that there were eruptions of bubbles during the 1980:1-1989:1 and 1989:2-2000:8 periods, but there was no explosive bubble during the 2000:9-2007 period. These findings are at variance with our Weibull test results. It should be, however, noted that the unit-root and cointegration test results seem to be inconsistent with some recent findings. Several recent studies have found that a bubble was detected only when the recent stock price series was included in the sample.

<Table 5> AEG Test for Conintegration between Stock Prices and Dividends

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Period | 1980:1-2007:10 | 1980:1-1987:10 | 1987:11-2000:8 | 2000:9-2007:10 | 1980:1-2007:10 | 1980:1-1987:10 | 1987:11-2000:8 | 2000:9-2007:10 |
| AEG | -0.732 (0.969) | -2.867 (0.178) | -1.732 (0.732) | -2.901 (0.167) | -1.815 (0.695) | -2.527 (0.315) | -0.528 (0.981) | -4.911 (0.001) |

*Numbers in parentheses denote p-values.

## 5. Concluding Remarks

The purpose of this study is to establish theoretical foundations for rational bubbles and to provide empirical evidence on the existence of rational bubbles in the U.S. stock market. There has recently been a resurgence of interest in bubbles in stock prices as U.S. stock prices have begun to rise steeply and exhibited wide fluctuations over the past 20 years or so. The bubble models elaborated by Hamilton and Whiteman (1985), Diba and Grossman (1988a,b,c), Froot and Obstfeld (1991) and others represent a significant departure from the conventional view in that they reinterpret rational bubbles in terms of market fundamentals. The attractive feature of the intrinsic bubble specification may be found in its ability to derive testable implications for bubbles by investigating the stationarity properties of stock prices and dividends or by parameterizing a specific bubble relationship as a function of market fundamentals.

However, the existing approach to modeling intrinsic bubbles still remains unsatisfactory. As Evans and others indicate, unit-root and cointegration tests are unable to detect an important class of rational bubbles. Alternative strategies to the traditional stationarity tests are found in Rappoport and White (1991) and Abreu and Brunnermeier (2003) who have not relied on unit-root and cointegration tests, and instead looked at the nature of bursting bubbles from the aspect of overreaction and informational errors.

In this study we formulate an information error model that allows one to derive bubble measures in a straightforward manner. This study provides a new method of testing for bubbles by specifying bubble measures in the context of the Weibull distribution. In our model, a bubble can occur as the result of overreaction of market participants to new information. To the best of our knowledge, this study is the first attempt to apply the Weibull distribution to the test of rational bubbles.

We have tested for the presence of bubbles in the U.S. stock market using monthly data from January 1980 to October 2007. We have divided the entire sample into three sub-samples: (1)

1980:1–1987:10, (2) 1987:11–2000:8, and (3) 2000:9-2007:10. The division of the sample period into three sub-sample periods roughly coincides with the uphill surge of stock prices followed by their downturns over the past three decades or so.

Our empirical analysis reveals that there is no evidence for extraneous bubbles in the U.S. stock market in any sample period. Our study further shows that there is no evidence in favor of the existence of intrinsic bubbles for the entire sample period as well as for the 1980:1–1987:10 and 1987:11–2000:8 sub-sample periods. However, it is worth noting that the surge and the subsequent collapse of stock prices during the 2000:9-2007:10 period turned out to be an explosive intrinsic bubble. These findings appear to be in agreement with some recent studies on rational bubbles in the U.S. stock market that found evidence for explosive bubbles when volatile stock prices since the middle of the 1990s are in the sample.

Interestingly, our unit-root and cointegration tests have produced opposite results. The stationarity tests show that there were explosive bubbles during the 1980:1-1989:1 and 1989:2-2000:8 periods, but there was no bubble during the 2000:9-2007 period. These findings are at variance with our Weibull test results. However, it should be noted that the stationarity test results seem to be inconsistent with some recent findings. Criticisms of unit-root and cointegration tests by Evans and others have been reinforced by this study.

## References

Abreu, Dilip and Markus K. Brunnermeier, Bubbles and Crashes, *Econometrica* 71, 2003, 173-204.

Ackert, L. and B. Smith, Stock Price Volatility, Ordinary Dividends, and Other Cash Flows to Shareholders, *Journal of Finance* 48, 1147-1160.

Bohl, Martin T., Periodically Collapsing Bubbles in the U.S. Stock Market? *International Review of Economics and Finance* 12, 2003, pp. 385-397.

Campbell, John Y. and Robert J. Shiller, Cointegration and Tests of Present Value Models, *Journal of Political Economy* 95, 1987, 1062-1088.

Campbell, John Y. and Robert J. Shiller, The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors, *Review of Financial Studies* 1, 1988, 195-228.

Charemza, Wojciech W. and Derek F. Deadman, Rational and Intrinsic Bubbles: A Reinterpretation of Empirical Results, *Applied Financial Economics* 5, 1995, 199-202.

Craine, Roger, Rational Bubbles: A Test, *Journal of Economic Dynamics and Control* 17, 1993, 829-846.

Crowder, B. and M. Wohar, Stock Price Effects of Permanent and Transitory Shocks, *Economic Inquiry* 36, 1998, 540-552.

Cunado, J., L.A. Gil-Alana, and F. Perez de Gracia, A Test for Rational Bubbles in the NASDAQ Stock Index: A Fractionally Integrated Approach, *Journal of Banking & Finance* 29, 2005, 2633-2654.

Cunado, J., L.A. Gil-Alana, and F.Perez de Gracia, Testing for Stock Market Bubbles Using Nonlinear Models and Fractional Integration, *Applied Financial Economics* 17, 2007, 1313-1321.

DeLong, David N. and Charles H. Whiteman, The Temporal Stability of Dividends and Stock Prices: Evidence from the Likelihood Function, *American Economic Review* 81, 1991, 600-617.

Diba, Behzad T. and Herschel I. Grossman, Explosive Rational Bubbles in Stock Prices? *American Economic Review* 78, 1988a, 520-529.

Diba, Behzad T. and Herschel I. Grossman, Rational Inflationary Bubbles, *Journal of Monetary Economics* 21, 1988b, 35-46.

Diba, Behzad T. and Herschel I. Grossman, The Theory of Rational Bubbles in Stock Prices, *Economic Journal* 98, 1988c, 746-754.

Engle, R., Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica* 50, 987-1007.

Engle, R., D. Lilien, and R. Robins, Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model, *Econometrica* 55, 391-407.

Evans, George W., Pitfalls in Testing for Explosive Bubbles in Asset Prices, *American Economic Review* 81, 1991, 922-930.

French, Kenneth R., G. William Schwert, and Robert F. Stambaugh, Expected Stock Returns and Volatility, *Journal of Financial Economics* 19, 1987, 3-29.

Froot, Kenneth A. and Maurice Obstfeld, Intrinsic Bubbles:The Case of Stock Prices, *American Economic Review* 81, 1991, 1189-1214.

Grossman, Sanford J. and Robert J. Shiller, The Determinants of the Variability of Stock Market Prices, *American Economic Review* 71, 1981, 222-227.

Hamilton, James D., On Testing for Self-fulfilling Speculative Price Bubbles, *International Economic Review* 27, 1986, 545-552.

Hamilton, James D. and Charles H. Whiteman, The Observable Implications of Self-fulfilling Expectations, *Journal of Monetary Economics* 16, 1985, 353-373.

Koustas, Zisimos and Apostolos Serletis, Rational Bubbles or Persistent Deviations from Market Fundamentals? *Journal of Banking And Finance* 29, 2005, 2523-2539.

Lamont, O., Earnings and Expected Returns, *Journal of Finance* 53, 1998, 1563-1587.

Nasseh, Alireza and Jack Strauss, Stock Prices and the Dividend Discount Model: Did Their Relation Break Down in the 1990s? *Quarterly Review of Economics and Finance* 44, 2004, 191-207.

Phillips, Peter C.B., Time Series Regression with a Unit Root, *Econometrica*, 55, 1987, 277-301.

Phillips, Peter C.B. and Pierre Perron, Testing for a Unit Root in Time Series Regression, *Biometrika* 74, 1988, 535-547.

Rappoport, Peter and Eugene N. White, Was There a Bubble in the 1929 Stock Market? *NBER Working Paper* No. 3612, 1991.

Sarno, L. and M.P. Taylor, An Empirical Investigation of Asset Price Bubbles in Latin American Emerging Financial Markets, *Applied Financial Markets* 13, 2003, 635-643.

Shiller, Robert J., Do Stock Prices Move Too Much To Be Justified by Subsequent Changes in Dividends? *American Economic Review* 71, 1981a, 421-436.

Shiller, Robert J., The Use of Volatility Measures in Assessing Market Efficiency, *Journal of Finance* 36, 1981b, 291-311.

Summers, Lawrence H., Does the Stock Market Rationally Reflect Fundamentals? *Journal of Finance* 41, 1986, 591-601.

Timmermann, A., Cointegration Tests of Present value Models with a Time-Varying Discount Factor, *Journal of Applied Econometrics* 10, 1995, 17-31.

Yuhn, Ky H., Stock Price Volatility: Tests for Linear and Nonlinear Cointegration in the Present Value Model of Stock Prices, *Applied Financial Economics* 6, 1996, 487-494.

# Was There an Explosive Bubble in U.S. Stock Prices before the Recent Stock Market Crash?

**Macroeconomics**
Prof. Ky-hyang Yuhn

# 1. Introduction

-The U.S. stock market has been increasingly volatile in recent years.

-There have been many episodes in history that may be adequately characterized as 'irrational exuberance' coined by former Federal Reserve chairman Alan Greenspan.

-Bubbles can be divided into rational and irrational bubbles, and extraneous and intrinsic bubbles.

## 1. Introduction

-Earlier views on a bubble ascribe it to some psychological factors such as herd behavior, animal spirits or cognitive biases in which bubbles propagate themselves (= extraneous bubble).

-It has been widely believed that a bubble, especially an extraneous bubble does not lend easily itself to direct testing: Evans (1991), Flood and Hodrick (1990), etc.

# 1. Introduction

-Economists traditionally used indirect methods to test for the presence of bubbles :Shiller's (1981) variance bounds tests.

-However, recent bubble models elaborated by Hamilton and Whiteman (1985), Diba and Grossman (1988a,b,c), Evans (1991), Froot and Obstfeld (1991), and others reinterpret rational bubbles in terms of market fundamentals (= intrinsic bubbles).

# 1. Introduction

-A number of recent studies have investigated the existence of intrinsic bubbles in stock markets by examining the stationarity properties of stock prices and dividends on the basis of unit-root and cointegration tests: Campbell and Shiller (1987, 1988), Diba and Grossman (1988), Crowder and Wohar (1988), Froot and Obstfeld (1991), Craine (1993), Charemza and Deadman (1995), Timmermann (1995), Lamont (1998), Bohl (2003), Sarno and Taylor (2003), Nasseh and Strauss (2004), Koustas and Serletis (2005), Cunado, Gil-Alana, and de Gracia (2005, 2007), among others.

# 1. Introduction

-However, unit-root approaches are unable to detect an important class of rational bubbles: Evans, Charemza and Deadman (1995), Ackert and Smith (1993).

-Rappoport and White (1991) have directly extracted an estimate of the path of the bubble and its probability of bursting.

-Abreu and Brunnermeier (2003) have also provided a setting in which new events can have a disproportionate impact relative to their intrinsic informational content without reference to the stationarity processes of stock prices and market fundamentals.

# 1. Introduction

-The main purpose of this study is to derive a bubble measure from the information error model and test whether the bubble measure is explosive using a Weibull distribution.

-We have tested for the existence of explosive bubbles in the U.S. stock market using monthly data from Jan. 1980 to Oct. 2007: 1980:1–1987:10, 1987:11–2000:8, and 2000:9–2007:10. Our empirical analysis shows that only the surge of stock prices peaking in October 2007 contained an explosive bubble.

## 2. A Review of the Literature

-Shiller (1981) has shown that the violation of the variance bounds implied by the present value model is overwhelming for the U.S. data. The violation of the variance bounds is interpreted as rejection of the efficient markets hypothesis.

-Hamilton and Whiteman (1985) have dealt with bubbles that can be dependent on market fundamental values for the first time and discussed bubbles from the perspective of the stationarity properties of stock prices and market fundamentals.

## 2. A Review of the Literature

-Diba and Grossman (1988) postulate that if there is no bubble premium in stock prices, stock prices should be cointegrated with market fundamentals in a nonlinear fashion. They have conducted cointegration tests for the S&P 500 for the period 1871-1986 and found that the U.S. stock prices do not contain explosive rational bubbles.

-Campbell and Shiller (1987) have tested for cointegration between stock prices and dividends using annual data for the S&P 500 index from 1871 to 1986, and found persistent deviations of stock prices from the present value model, which can be interpreted as evidence for the presence of rational bubbles in U.S. stock prices.

## 2. A Review of the Literature

-Evans (1991) has shown that the cointegration of stock prices with dividends cannot be viewed as evidence against the presence of bubbles. His simulation tests show that over the sample, there appear to be four bubble eruptions, each followed by a collapse.

-Froot and Obstfeld (1991) have tested for a unit root in the price-dividend ratio (P/D) using the S&P 500 index for the period 1900-1988. Using the Phillips-Perron (1988) unit-root tests, they were unable to reject the unit root null hypothesis in five of six cases, which indicates the lack of cointegration between stock prices and dividends. They have concluded that deviations from the PV model are consistent with the presence of rational bubbles in stock prices.

# 2. A Review of the Literature

-Craine (1993) has applied the augmented Dickey-Fuller (1981) unit-root test to the price-dividend ratio (P/D) using annual S&P 500 data from 1876 to 1988. His test results show that either the price-dividend ratio contains a rational bubble or the discount factor must be stochastic and contain a large predictable component.

-Yuhn (1997) has derived a dynamic form of cointegration between stock prices and dividends. His empirical results reveal that little evidence for linear cointegration is found, but the evidence of nonlinear cointegration is overwhelming for U.S. monthly data from 1959:1 to 1992:6, indicating no volatility in the U.S. stock market during the sample period.

# 2. A Review of the Literature

-Bohl (2003) has found that there were no periodically collapsing bubbles in the U.S. stock market over the 1871-1995 period, but the sample including the rapid price increases since the middle of the 1990s (1871-2001) is interpretable in favor of the existence of collapsing bubbles in U.S. stock prices.

-Nasseh and Strauss (2004) have discovered that stock prices are overvalued by 43% during the late 1990s. They have found that declines in nominal interest rates explain approximately half of the overvaluations, and a break in dividend payments in the mid-1990s can explain the remaining stock price overvaluation.

## 2. A Review of the Literature

-Koustas and Serletis (2005) have examined the behavior of the dividend-price ratio (D/P). They have maintained that the presence of a unit root in the log dividend yield (d – p) is consistent with rational bubbles in stock prices. Their test rejects the unit-root null hypothesis, strongly suggesting that the log dividend yield is mean reverting.

-Cunado et.al. (2005) have investigated whether there is any fractional cointegration between the NASDAQ index and its corresponding dividend yield. Using the dividend-price ratio (D/P) model, they have found the order of fractional cointegration between the two variables to be greater than 0 but less than 1, which leads them to reject the null hypothesis of rational bubbles for daily and weekly data. However, they have failed to reject the null hypothesis for monthly data.

## 2. A Review of the Literature

-Cunado et. al. (2007) have tested for the existence of bubbles in the S&P 500 index using fractional integration techniques. They have found the order of integration for the log price-dividend ratio to be equal to or higher than 1 and concluded that there exists a stock market bubble in the S&P 500 index over the entire period.

-Unlike the existing studies that are primarily concerned with the stationarity properties of stock prices and dividends, Rappoport and White (1991) have used the behavior of the premia demanded on loans collateralized by the purchase of stocks as a bubble measure and found that although standard tests find no bubbles in the stock price data for the last 100 years, their test points to the existence of a bubble during the Great Depression period of 1928-29.

## 2. A Review of the Literature

-Abreu and Brunnermeier (2003) have developed a dynamic game model in which bubbles can persist even though all rational arbitrageurs know that the stock price is too high, and they jointly have the ability to correct the mispricing. Their model provides a setting in which “overreaction” and self-feeding price processes lead to full-fledged crashes.

-Our model shares some common theme with Rappoport and White (1991) and Abreu and Brunner (2003) in that our model extracts a bubble measure as market participants’ overreaction to the new information on market fundamentals.

## 3. The Theoretical Model

-The present value (PV) model of stock prices:

(1) $P_t = \delta E_t(P_{t+1} + D_{t+1})$

-The solution to equation (1):

(2) $P_t = \sum_{k=1}^{\infty} \delta^k E_t(D_{t+k}) + \lim_{T \to \infty} E_t \delta^T P_{t+T}$

-If we impose a transversality condition on equation (2), then we obtain the unique solution:

(3) $P_t = \sum_{k=1}^{\infty} \delta^k E_t(D_{t+k})$

# 3. The Theoretical Model

-If the transversality condition fails to hold, we have a bubble part:

(4) $$P_t = \sum_{k=1}^{\infty} \delta^k E_t(D_{t+k}) + B_t$$

where  measures the bubble term and satisfies the following process:

(5) $$B_t = \delta E_t B_{t+1}$$

## 3. The Theoretical Model

-Hamilton (1986) and Diba and Grossman (1988a) reformulate the PV model in a different way from the conventional:

(6)

$$P_t = \delta(D_{t+1} + E_t P_{t+1} + \pi_t)$$

where $\pi$ is a catch-all random variable that is not observed by researchers, but can be observed only by market participants. $\pi$ is assumed to be stationary.

-Diba and Grossman(1988a) have conducted an empirical investigation of the Hamilton model using a slightly modified version.

(7)

$$P_t = \delta E_t (P_{t+1} + \alpha D_{t+1} + \pi_{t+1})$$

## 3. The Theoretical Model

-Diba and Grossman have proposed that if the unobserved is stationary, and first differenced dividends and first-differenced stock prices are stationary, there does not exist a bubble. That is, there is no bubble if is I(0), and stock prices and dividends are I(1), and stock prices and dividends are cointegrated (CI(1,1)).

# 3. The Theoretical Model

-Diba and Grossman have derived the following estimation equation:

(8) $P_{t+1} + \gamma D_{t+1} - (1/\delta)P_t = e_{t+1} - \pi_{t+1}$

where $e_{t+1} = P_{t+1} + \gamma D_{t+1} + \pi_{t+1} - E_t\left(P_{t+1} + \gamma D_{t+1} + \pi_{t+1}\right)$

The LHS becomes stationary if $\pi_{t+1}$ is stationary, since $e_{t+1}$ is not serially correlated.

## 3. The Theoretical Model

-In this study, we develop a rational bubble model in the spirit of Hamilton (1986) and Diba and Grossman (1988) without introducing unobservable random variables.

(9) $$P_{t-1} = \sum_{k=1}^{\infty} \delta^{k} E_{t-1}(D_{t+k-1})$$

(10) $$P_{t-1} - \delta P_{t} = E_{t-1}\delta D_{t} - \sum_{k=1}^{\infty} \delta^{k+1}[E_{t} - E_{t-1}]D_{t+k}$$

(11a) $$\delta(P_{t} + D_{t}) - P_{t-1} = \sum_{k=0}^{\infty} \delta^{k+1}[E_{t} - E_{t-1}]D_{t+k}$$

(11b) $$P_{t} + D_{t} = \frac{1}{\delta}P_{t-1} + \sum_{k=0}^{\infty} \delta^{k}[E_{t} - E_{t-1}]D_{t+k}$$

## 3. The Theoretical Model

-Equation (11b) can be further simplified as

(12) $P_t = (1+r)P_{t-1} - D_t + \nabla_t$

where $\nabla_t = \sum_{k=0}^{\infty} \delta^k [E_t - E_{t-1}] D_{t+k}$ denotes the present value of the sum of the forecast errors of market participants. (information error model). The term, $\nabla_t$, is used as a measure of bubbles since an intrinsic bubble is an overreaction of participants to new information on market fundamentals.

## 3. The Theoretical Model

- $\nabla_t$ is the same as the unobserved variable in the Hamilton-Whiteman and Diba-Grossman models.

(13) $P_{t+1} = (1/\delta) P_t - E_t D_{t+1} + v_{t+1}$

where $v_{t+1} = \sum_{k=1}^{\infty} \delta^k [E_t - E_{t-1}] D_{t+k+1}$

Let $E_{t+1} D_{t+1} - E_t D_{t+1} = e_{t+1}$

(14) $P_{t+1} + D_{t+1} - (1/\delta) P_t = e_{t+1} + v_{t+1}$

## 3. The Theoretical Model

This expression is the same as the Diba-Grossman model when = 1 in their model. We note that $\nabla_{t+1} = e_{t+1} + v_{t+1}$ ($\nabla_{t+1} = e_{t+1} + \pi_{t+1}$ in the Diba-Grossman model). Thus, we can extract bubbles from $P_t + D_t - (1/\delta) P_{t-1}$ without introducing arbitrarily an unobserved random variable.

## 3. The Theoretical Model

-The bubble measure ($\nabla_t$) is assumed to have a Weibull distribution. We will denote a bursting bubble by a continuous random variable $T$. If $T$ has a probability density function $f(t)$, then the probability that optimistic expectations about stock prices continue to hold until a specific time $t$ is given by(15) where $z(t)$ is the survival function for optimistic forecasting until a specific time $t$, and $F(t)$ is the cumulative density function.

## 3. The Theoretical Model

-The likelihood of the bursting of a bubble in the next small unit of time $\Delta t$, given that a bubble has survived until time t.

(16) $\theta(t) = \lim_{\Delta t \to 0} \frac{[F(t+\Delta t) - F(t)]}{\Delta t} \cdot \frac{1}{z(t)} = \frac{F'(t)}{z(t)} = \frac{f(t)}{z(t)} = \frac{f(t)}{1 - F(t)}$

(17) $\theta(t) = -z'(t)/z(t) = -\, d \ln z(t)/dt$

## 3. The Theoretical Model

-The probability density function (*pdf*) is

(18a) $\ln z(t) = -\int \theta(t)dt + \ln c$

(18b) $z(t) = c \exp(-\int \theta(t)\, dt)$

-The cumulative density function can be expressed as

(19) $F(t) = 1 - ce^{-\int \theta(t)\, dt}$

## 3. The Theoretical Model

-If *f*(*t*) follows the Weibull distribution, then we have the following *pdf*

(20) $f(t) = \alpha\lambda t^{\alpha-1}\exp(-\lambda t^{\alpha})$

and the bursting rate is given by

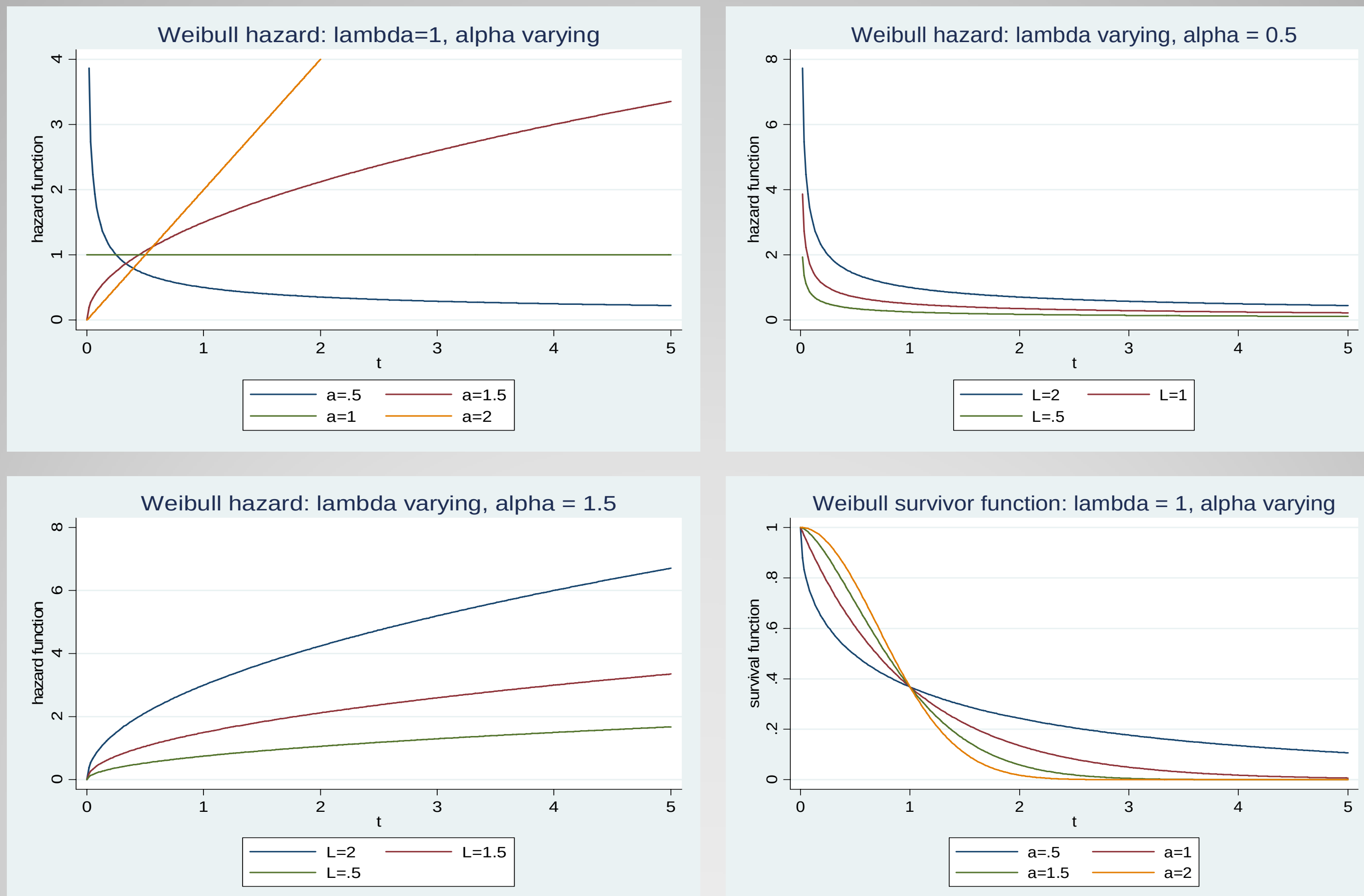
(21) $\theta(t) = \alpha\lambda t^{\alpha-1}$

-When λ is equal to 1, we have , which gives the possibility of an extraneous bubble (speculative bubble).

### 3. The Theoretical Model

-If $\alpha$ is smaller than one, the bursting rate decreases, so that the possibility of bursting will also decrease as time goes on.
-If $\alpha$ is equal to one, the bursting rate is constant, so that the possibility of bursting will also be constant. This case is the same as the well-known exponential distribution.
-If $1 < \alpha < 2$, the bursting rate increases at a decreasing rate, so that the possibility of bursting also will increase at a decreasing rate.
-If $\alpha$ is greater than 2, the bursting rate increases at an increasing rate, so that the possibility of bursting will increase at an increasing rate. Thus, an explosive bubble occurs when $\alpha$ is greater than 2.

# 3. The Theoretical Model



## 3. The Theoretical Model

-We obtain an intrinsic bubble model if we let  and an extraneous bubble model if we let $\lambda$ = 1:

(22a) $\theta(t) = \alpha t^{\alpha-1}\exp(X'\beta) = \psi_1(t)\psi_2(X')$ :Intrinsic bubbles

(22b) $\theta(t) = \alpha t^{\alpha-1} = \psi_1(t)$ with $\psi_2(X') = 1$ :Extraneous bubbles

where *X* denotes the set of variables that are thought to be related to a firm's fundamental values.

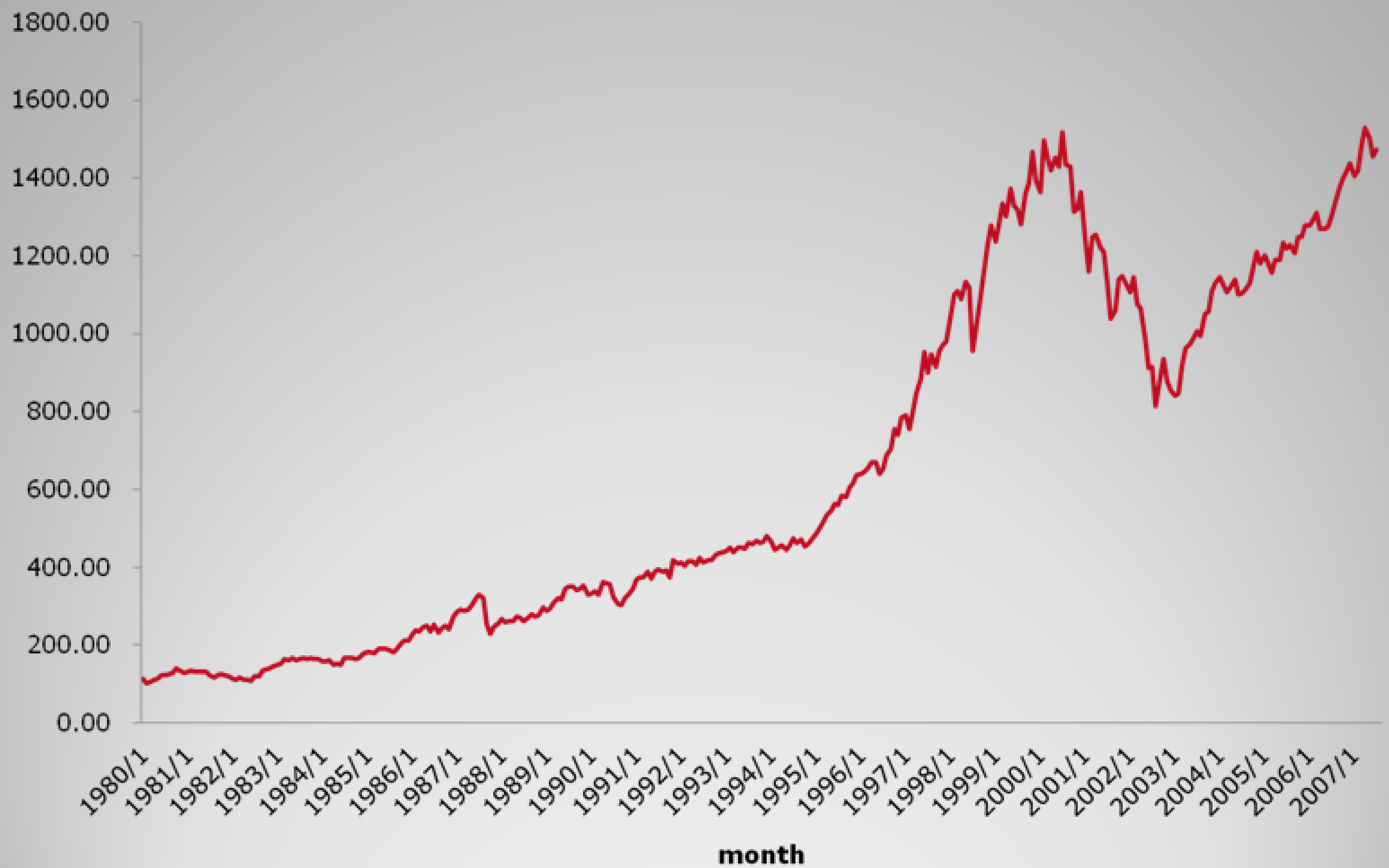
# 4. Empirical Analysis

## 4.1. Data Description

-We have used U.S. monthly data from 1980:1 to 2007:10. The sample is divided into three sub-periods:1980:1-1987:10(period 1), 1987:11-2000:8(period 2), and 2000:9-2007:10(period 3).

-The data used in this study include the S&P 500 composite index, the price-earnings ratio (PER), the term spread (TERM) between the short-term interest rate (3-month T-bill rate) and the long-term interest rate (10-year T-bond rate), the default rate (DEF), the nominal effective exchange rate (EX), and the unemployment rate (UNEMP).

# 4. Empirical Analysis

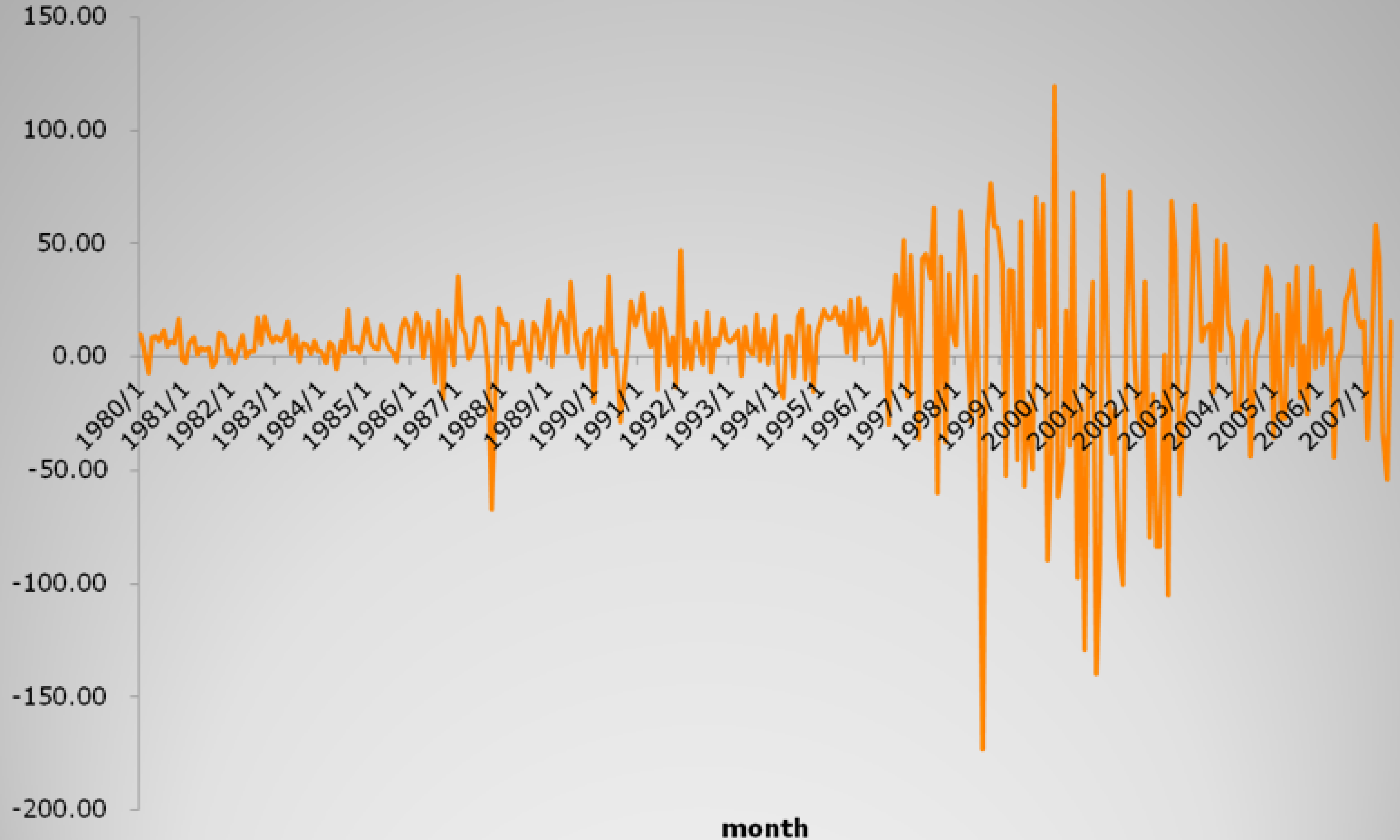


# 4. Empirical Analysis

## 4.2. Measuring Bubbles

Our information error model enables us to divide current stock prices into two parts, the market fundamental measure that is given by  and the bubble measure that deviates from the expected orbit of the information error model.

# 4. Empirical Analysis



# 4. Empirical Analysis

## 4.3. Empirical Results

### A. Unit-Root Tests

-We have conducted unit-root tests for the following data-generating processes.

(23a) $y_t = \mu + \alpha y^*_{t-1} + u^*_t$

(23b) $y_t = \mu + \beta(t - T/2) + \alpha y_{t-1} + u_t$

# 4. Empirical Analysis

<Table 1> Unit-Root Tests

| Variables | Phillips-Perron Test | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $y_t = \mu + \alpha y_{t-1} + u_t$ | | | | $y_t = \mu + \alpha y_{t-1} + \beta(t - T/2) + u_t$ | | | |
| | 1980:1-2007:10 | 1980.1-1987.10 | 1987.11-2000.8 | 2000.9-2007.10 | 1980:1-2007:10 | 1980.1-1987.10 | 1987.11-2000.8 | 2000.9-2007.10 |
| Bubble Measure | -18.22** | -6.30** | -15.20** | -7.86** | -18.29** | -6.33** | -15.62** | -8.69** |
| PER | -2.15 | -1.11 | -0.93 | -1.53 | -2.33 | -2.25 | -1.87 | -2.34 |
| Term Premium | -4.59** | -3.47** | -2.41 | -1.43 | -4.54** | -3.78** | -2.39 | -2.20 |
| Unemployment Rates | -1.58 | -1.36 | -0.32 | -2.00 | -2.81 | -2.05 | -1.17 | -2.52 |
| Default Rates | -2.10 | -1.34 | -1.46 | -0.24 | -1.85 | -2.47 | -1.39 | -2.62 |
| Exchange Rates | -2.35 | -1.51 | -1.14 | -0.34 | 0.94 | -0.42 | -2.88 | -3.02 |

*indicates significance at the 1% level and ** significance at the 5% level.

# 4. Empirical Analysis

-The tests have produced actually the same results for (23a) and (23b).

(1) The bubble measure is unambiguously stationary. The bubble measure has turned out to be stationary in both the data-generating processes in every sub-period as well as in the entire sample period.

(2) The term spread is stationary only in period 1 and in the entire period.

(3) We fail to reject the unit root null hypothesis for all other variables.

(4) Thus, all other variables except for the bubble measure (in all sample periods) and the term spread (in period 1 and the entire period) contain a unit root.

# 4. Empirical Analysis

## B. Tests for Bubbles: The Weibull Tests

## B.1. Testing for Purely Extraneous Bubbles

<Table 2> Tests for Extraneous Bubbles

| Period | 1980:1-2007:10 | 1980:1-1987:10 | 1987.11-2000:8 | 2000:9-2007:10 |
|---|---|---|---|---|
| $\alpha$ | 1.32* | 1.37* | 1.48* | 1.64* |

* indicates significance at the 5 percent level.

## 4. Empirical Analysis

-There is no evidence for extraneous bubbles in the U.S. stock market in all samples, indicating that there is no possibility of seemingly too high stock prices to become explosive bubbles driven by a wave of people's psychological biases or fads. The estimated value of $\alpha$ is 1.32 for the entire sample period, and 1.37 for sample period 1, 1.48 for sample period 2, and 1.64 for sample period 3. All the estimates are significant at the 5 percent level of significance.

## 4. Empirical Analysis

-We can conclude that although stock prices seem to have been overvalued on certain time intervals, overheated stock prices were not driven by extraneous factors.

# 4. Empirical Analysis

## B. 2. Testing for Intrinsic bubbles

-We have included a set of variables in the hazard model that affect stock prices. These market fundamental variables include the PER (price-earnings ratio), TERM (term spread), UNEMP (unemployment rate), DEF (default rate) and EX (exchange rate).

# 4. Empirical Analysis

<Table 3> Tests for Intrinsic Bubbles

| | With the results of the unit root test | | | |
|---|---|---|---|---|
| Period | 1980:1-2007:10 | 1980:1-1987:10 | 1987:11-2000:8 | 2000:9-2007:10 |
| $\alpha$ | 1.52* | 1.78* | 1.61* | 1.92* |
| $\beta_0$ | 0.80 | -0.39 | 1.09 | -9.14* |
| PER | 0.06* | -0.05 | 0.01 | -0.13* |
| TERM | 0.04 | -0.05 | 0.34* | 0.13 |
| UNEMP | -0.36* | -0.39* | -0.49* | 0.21 |
| DEF | 0.11* | -0.18 | -0.01 | 0.34 |
| EX | -0.02* | 0.06* | -0.01 | 0.08 |

* indicates significance at the 5 percent level.

# 4. Empirical Analysis

(1) The value of α is 1.52 for the entire sample period, and the coefficient is significant. Thus, no evidence in favor of explosive bubbles is found for the entire sample period.

(2) The value of α is considerably less than 2 at the 5 percent level of significance in both the sample period 1 (1980:1-1987:10) and sample period 2 (1987:11-2000:8). Thus, during these briefer sub-sample periods, stock prices seem to have risen too much, but they did not develop into explosive bubbles.

# 4. Empirical Analysis

(3) The value of $\alpha$ is 1.92 for the sample period 3 (2000:9-2007:10) and significant at the conventional level of significance. This finding can be construed as evidence for the existence of an explosive bubble during the 2000:9-2007:10 period. As observed by Rappoport and White (1991), it is entirely possible to detect a bubble in briefer sub-sample periods even though bubbles may not be present over a longer period of time that encompasses the shorter sub-sample periods.

(4) We have tested for the model using a broader pool of market fundamentals including industrial production and earnings per share, but this extension has little effect on the test results.

# 4. Empirical Analysis

## C. Tests for Bubble: Unit-Root and Cointegration Tests

<Table 4> Phillips-Perron Unit-Root Test for Prices (P) and Dividends (D)

| | $y_t = \mu + \alpha y_{t-1} + u_t$ | | | | $y_t = \mu + \alpha y_{t-1} + \beta (t - T/2) + u_t$ | | | |
|---|---|---|---|---|---|---|---|---|
| Period | 1980:1-2007:10 | 1980:1-1987:10 | 1987:11-2000:8 | 2000:9-2007:10 | 1980:1-2007:10 | 1980:1-1987:10 | 1987:11-2000:8 | 2000:9-2007:10 |
| P | 0.129 (0.974) | -0.753 (0.827) | 3.152 (1.000) | -1.081 (0.720) | -1.912 (0.646) | -2.262 (0.450) | -0.184 (0.993) | -2.928 (0.159) |
| D | 3.395 (1.000) | 1.179 (0.998) | -2.117 (0.238) | 1.256 (0.998) | 1.386 (1.000) | -3.077 (0.118) | -6.013 (0.000) | -2.205 (0.325) |
| ΔP | -18.716 (0.000) | -6.584 (0.000) | -14.319 (0.000) | -8.334 (0.000) | -18.732 (0.000) | -6.329 (0.000) | -16.819 (0.000) | -8.798 (0.000) |
| ΔD | -25.865 (0.000) | -18.682 (0.000) | -24.066 (0.000) | -12.218 (0.000) | -28.433 (0.000) | -13.910 (0.000) | -25.953 (0.000) | -14.604 (0.000) |

*Numbers in parentheses denote p-values.

## 4. Empirical Analysis

-Our Phillips-Perron unit-root tests reveal that stock prices unambiguously contain a unit root in both the processes for all sample periods, and the results for dividends are very similar to those of stock prices only with the exception that the dividend series (with both a constant and a trend) is stationary during sample period 2. On the other hand, first-differenced stock prices and first-differenced dividends are stationary in all data-generating processes for all samples.

## 4. Empirical Analysis

-We have tested for cointegration between stock prices and dividends using the augmented Engle-Granger test. We are unable to reject the unit-root null hypothesis for the entire sample period as well as for sub-sample periods 1 and 2, but we reject the unit-root null hypothesis for sub-sample period 3.

## 4. Empirical Analysis

-According to the unit-root and cointegration test results, we should conclude that there were eruptions of bubbles during the 1980:1-1989:1 and 1989:2-2000:8 periods, but there was no explosive bubble during the 2000:9-2007 period. These findings are at variance with our Weibull test results.

# 4. Empirical Analysis

<Table 5> AEG Test for Conintegration between Stock Prices and Dividends

| | $y_t = \mu + \alpha y_{t-1} + u_t$ | | | | $y_t = \mu + \alpha y_{t-1} + \beta(t - T/2) + u_t$ | | | |
|---|---|---|---|---|---|---|---|---|
| Period | 1980:1-2007:10 | 1980:1-1987:10 | 1987:11-2000:8 | 2000:9-2007:10 | 1980:1-2007:10 | 1980:1-1987:10 | 1987:11-2000:8 | 2000:9-2007:10 |
| AEG | -0.732 (0.969) | -2.867 (0.178) | -1.732 (0.732) | -2.901 (0.167) | -1.815 (0.695) | -2.527 (0.315) | -0.528 (0.981) | -4.911 (0.001) |

*Numbers in parentheses denote p-values.

## 5. Concluding Remarks

-As Evans and others indicate, unit-root and cointegration tests are unable to detect an important class of rational bubbles. Alternative strategies to the traditional stationarity tests are found in Rappoport and White (1991) and Abreu and Brunnermeier (2003) who have not relied on unit-root and cointegration tests, and instead looked at the nature of bursting bubbles from the aspect of overreaction and informational errors.

## 5. Concluding Remarks

-This study provides a new method of testing for bubbles by specifying bubble measures in the context of the Weibull distribution. In our model, a bubble can occur as the result of overreaction of market participants to new information. To the best of our knowledge, this study is the first attempt to apply the Weibull distribution to the test of rational bubbles.

## 5. Concluding Remarks

-We have tested for the presence of bubbles in the U.S. stock market using monthly data from January 1980 to October 2007: (1) 1980:1–1987:10, (2) 1987:11–2000:8, and (3) 2000:9-2007:10. The division of the sample period into three sub-sample periods roughly coincides with the uphill surge of stock prices followed by their downturns over the past three decades or so.

-There is no evidence for extraneous bubbles in the U.S. stock market in all sample periods.

## 5. Concluding Remarks

-There is no evidence in favor of the existence of intrinsic bubbles for the entire sample period as well as for the 1980:1–1987:10 and 1987:11–2000:8 sub-sample periods. However, our analysis shows that the surge and the subsequent collapse of stock prices during the 2000:9-2007:10 period turned out to be an explosive intrinsic bubble.

## 5. Concluding Remarks

-Our unit-root and cointegration tests have produced opposite results. The stationarity tests show that there were explosive bubbles during the 1980:1-1989:1 and 1989:2-2000:8 periods, but there was no bubble during the 2000:9-2007 period.